Robust control for a class of interval model: application to the force control of piezoelectric cantilevers

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***Abstract*— A method to design robust performances controllers is proposed in this paper. The method is valuable for a class of uncertain parametric systems: systems with zero-order numerator.**

In this work, interval model that accounts parametric uncertainties is described using interval analysis. Robust controller is derived by combining the interval arithmetic with the classical direct synthesis. As the derived controller is also an interval, we prove using numerical analysis that the midpoint can be taken as the final controller to be implemented. The proposed design method is applied to the control of manipulation force in piezocantilevers where the compliance of the manipulated objects is uncertain. The experimental results show the efficiency of the proposed method.

***Index Terms*— Robust performances, Interval models, controller design, piezocantilevers, manipulation force.**

I. INTRODUCTION

Smart materials are widely used in the development of microsystems and microrobots. Piezoelectric materials are among the most appreciated because of their high resolution and rapidity [1]. One of the main applications of piezoelectric materials in microsystems is piezoelectric microgrippers dedicated to micromanipulation and microassembly tasks [2][3]. A piezoelectric microgripper is composed of two piezocantilevers Fig. 1. They are used to pick, transport and place micro-objects with high positioning accuracy.

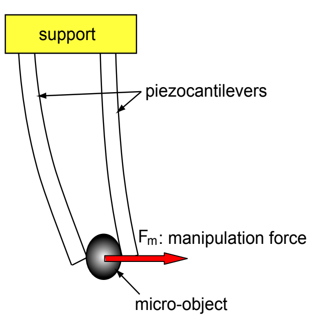

Fig. 1. A microgripper manipulating a micro-object.

During the manipulation of micro-objects, especially biological and optical materials that are usually fragile, pure position control is usually not adequate. Force control is often needed to prevent damage of the manipulated micro-objects and also to well keep them in the microgripper. For that, the first piezocantilever of the microgripper must be controlled on position while the second one on force [4][5].

In order to synthesize a controller for the manipulation force, a model is necessary. However, it is known that the model linking this manipulation force and the input control is dependent on the micro-object characteristics [4]. Therefore, it is necessary to have a model that accounts the micro-objects characteristics. In many cases, such characteristics are known only in a range of variation. A controller must be therefore computed to ensure robust performances for all micro-objects characterized inside this range.

To ensure robust perfomances for uncertain systems, robust controllers synthesis which accounts model uncertainties is needed. Among these controllers, we find the robust control laws H_2, μ-synthesis and H_∞ that have been used to control microsystems, examples are given in [6] [7]. The efficiency of these control methods is proved in several applications while its major disadvantage is the derivation of high-order controllers which are time consuming and limit the embedding possibilities. A possible alternative to these classical robust control laws is to combine classical techniques of linear control design with interval analysis to construct simple robust controllers. Indeed, contrarily to scalar points, intervals are values that represent a set of real numbers that can be used to bound parametric uncertainties.

The first idea on interval arithmetic has appeared in 1924 by Burkill and 1931 by Young, then later in 1966 with R.E. Moore's works [8]. Since, several applications on interval analysis have been raised. Many of them relates to guaranteed estimation and robust stability while few concerns the design of controllers ensuring performances. In [9], Jaulin and Walter have proposed guaranteed parameters estimation based on the SIVIA algorithm (Set Inversion Via Interval Analysis). The works in [10][11][12] utilize a linear model characterized by a bounded error equation (interval) in order to define the values set of parameters compatible with the measurements, model and error bounds. In [12][13], the stability analysis of the closed-loop with a given controller was proposed using the Routh's criteria and the Kharithonov's theorem. Concerning the design of controller, [14] proposed an approach of state feedback control combined with the intervals for the parameters model to synthesize a controller that ensures the stability. In [15], an approach of robust controller design based on the inclusion of the interval closed-loop transfer inside an interval transfer defining the wanted performances have been proposed. The

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computation of the controller is formulated as a set-inclusion problem, where a check of the inclusions satisfaction for a set of frequencies is needed. In [16], robust controller synthesis approach guaranteeing both the robust stability and performances for an interval system is addressed. In the latter work, the computation of two controllers is required, a robust stabilizing feedback controller is firstly computed. Subsequently, a pre-filter is constructed to ensure the wanted performances. [17] proposed a control algorithm prediction-based interval model and its application to a welding process.

The main objective of this paper is the design of a robust controller ensuring performances and valuable for systems with multiple uncertain parameters. The proposed method is valuable for a class of uncertain systems: systems with zero-order numerator. In addition to its principle simplicity, the main advantage is the derivation of a low order controller. For that, we combine direct controller synthesis with the interval analysis.

The paper is organized as follows. First the interval computation and its basic terms are presented. Afterwards, we introduce the direct synthesis method to compute an interval controller using the wanted performances and the interval system model. To prove the efficiency of the proposed method, an application on the force control of piezocantilevers has been presented. Finally, the experimental results end the paper.

II. Basic Terms and Concepts on Intervals

In this section, we remind interval arithmetic (see [8] for more details).

A. Definitions

A closed interval, denoted by $[x]$, is the set of real numbers given by:

$$[x] = [x^-, x^+] = \{x \in R / x^- \leq x \leq x^+\} \qquad (1)$$

The endpoints x^- and x^+ are respectively the left and right endpoint of $[x]$.

The width of an interval $[x]$ is given by:

$$w([x]) = x^+ - x^- \qquad (2)$$

The midpoint of [x] is given by:

$$mid([x]) = \frac{x^+ + x^-}{2} \qquad (3)$$

The radius of $[x]$ is defined by:

$$rad([x]) = \frac{x^+ - x^-}{2} \qquad (4)$$

B. Operations on intervals

The elementary mathematical operations are extended to intervals, the operation result between two intervals is an interval containing all the operations results of all pairs of numbers in the two intervals.

Given two intervals $[x] = [x^-, x^+]$, $[y] = [y^-, y^+]$ and $\circ \in \{+, -, ., /\}$, we can write that:

$$[x] \circ [y] = \{x \circ y \,|\, x \in [x], y \in [y]\} \qquad (5)$$

C. Interval system

Parametric Uncertain systems can be modeled by interval systems. An interval system denoted $[G](s, [p], [q])$ is a system where $[p]$ and $[q]$ are two boxes of interval numbers:

$$[G](s, [p], [q]) = \frac{\sum_{j=0}^{m} [q_j] s^j}{\sum_{i=0}^{n} [p_i] s^i} = \left\{ \frac{\sum_{j=0}^{m} q_j s^j}{\sum_{i=0}^{n} p_i s^i} \,\middle|\, \begin{array}{c} q_j \in [q_j^-, q_j^+] \\ p_i \in [p_i^-, p_i^+] \end{array} \right\} \qquad (6)$$

with: $[q] = [[q_1], ..., [q_n]]$ and $[p] = [[p_1], ..., [p_n]]$.

III. Computation of the Controller

The objectif of this section is to derive robust control law by combining the previous interval analysis with a linear theory of control called direct synthesis of controller. The proposed method is suitable for single-input single-output interval systems.

A. Closed-loop control

Given an uncertain system $[G](s, [a])$ controlled by a controller $[C](s)$ as shown in Fig. 2. Let $[H](s, [b])$ be the closed-loop transfer.

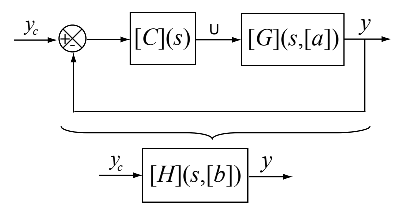

Fig. 2. A closed-loop system.

B. Definition of the system and of the wanted closed-loop model

In this work, we aim to design robust control law for a class of interval plants. Define the following interval system with zero-order numerator:

$$[G](s, [a]) = \frac{1}{\sum_{i=0}^{n} [a_i] s^i} \qquad (7)$$

Such as: $[a] = [[a_0], ..., [a_n]]$ is a box of uncertain parameters.

Remark 1: The effect of disturbances can be assumed to be in the range of interval in which parameter lies.

From given specifications (generally given in terms of settling time, static error, small overshoot,...etc), an interval closed-loop model denoted $[H](s, [b])$ is also derived. It is known as a reference interval model and is given by:

$$[H](s,[b]) = \frac{1}{\sum_{j=0}^{m}[b_j]s^j} \quad (8)$$

Where $[b] = [[b_0], ..., [b_m]]$ is a box of interval parameters.

C. Direct synthesis of the controller

The main idea of this method consists to compute a controller $[C](s)$ from the wanted closed-loop transfer $[H](s,[b])$ and from the interval model $[G](s,[a])$, the transfer $[H](s,[b])$ being derived from given specifications.

The closed-loop transfer is defined by:

$$[H](s,[b]) = \frac{[C](s)[G](s,[a])}{1+[C](s)[G](s,[a])} \quad (9)$$

From (9), we derive the interval controller $[C](s)$:

$$[C](s) = \frac{1}{[G](s,[a])\left(\frac{1}{[H](s,[b])}-1\right)} \quad (10)$$

As the controller contains $1/[G](s,[a])$, the method can be classified as a compensation technique.

Based on the interval arithmetic presented in section. II, the following interval controller is derived after introducing (8) in (10) and replacing $[G](s,[a])$ by (7):

$$[C](s,[a],[b]) = \frac{\sum_{i=0}^{n}[a_i]s^i}{\sum_{j=0}^{m}[b_j]s^j - 1} \quad (11)$$

The causality of the controller is obtained when:

$$m \geq n \quad (12)$$

Note that, there exist a set of controllers inside the interval controller defined in (11) that guarantee the required performances. The robustness of such controller to ensure performances depends on the wanted specifications and the interval width of the parameters of the uncertain system.

In the last section, using the SIVIA algorithm, we demonstrate that the midpoint controller is one among the possible solutions.

IV. Application to piezocantilevers

The aim of this section is to apply the proposed method for controlling the manipulation force of piezocantilevers on different micro-objects. Instead of manipulating micro-objects, we manipulate beams (cantilevers) with known compliances. A piezocantilever is a cantilever based on piezoelectric layers. Let (Fig. 3) shows a piezocantilever with rectangular cross-section and based one passive layer and one piezoelectric layer. It is also called unimorph piezocantilever. When a voltage U is applied to the piezolayer, it contracts or expands. As a result, the whole cantilever bends with a deflection δ.

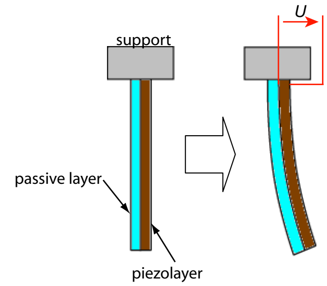

Fig. 3. Piezocantilever principle.

A. presentation of the setup

The experimental setup Fig. 4 is composed of:

- A unimorph piezocantilever with the dimensions $L \times b \times h = 15mm \times 2mm \times 0.3mm$,
- Two beams having different compliances, the first beam is rigid while the second one is flexible. The principle is shown in Fig. 4-a,
- a Keyence optical sensor with $10nm$ of resolution is used to measure the piezocantilever deflection. We utilize a computer-DSpace hardware combined with the Matlab-Simulink software for the implementation of the controller.

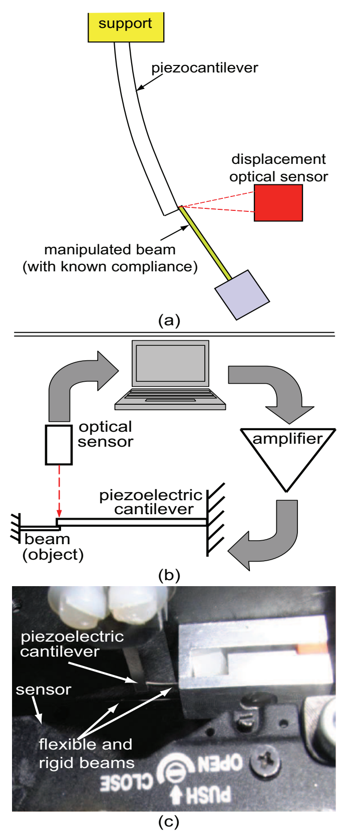

Fig. 4. The experimental setup.

B. Definition of the interval model

1) Modelling: According to the works in [4][5], the linear relation that relates the input voltage U applied to a piezocantilever, the force applied to the manipulated micro-object and the resulting deflection δ (see Fig. 5) can be written as follows:

$$\delta = (d_p.U - s_p.F_m).D(s) \tag{13}$$

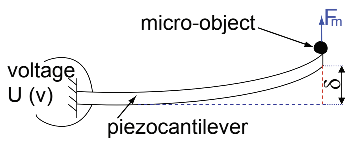

Fig. 5. A piezocantilever in contact with a micro-object.

where $s_p > 0$ is the elastic constant of the piezocantilever (in our case $s_p = 2\mu m/mN$), $d_p > 0$ is the piezoelectric constant and $D(s)$ is the transfer function representing the dynamic part of the piezocantilever (with $D(0) = 1$). $D(s)$ and d_p have to be identified.

Note that (13) is correct when the manipulator (piezocantilever) is in contact with the micro-object.

As demonstrated in [4], the relation between the manipulation force, the deflection and the characteristics of the micro-object can be written as follows:

$$\delta = s_o.F_m.D_o(s) \tag{14}$$

where $s_o > 0$ represents the micro-object compliance and $D_o(s)$ is its dynamic part.

In our work, we neglect the dynamic part of the micro-object, i.e $D_o(s) = 1$. This assumption is valuable when the compliance of the manipulated objects are not too high, and when its mass is negligible which is the case for micro-objects. Therefore we have:

$$\delta = s_o.F_m \tag{15}$$

Finally, after replacing the deflection δ in (13) by that of (15), we obtain the linear transfert voltage-force that takes into account the micro-object characteristics:

$$G(s) = \frac{F_m}{U} = d_p \frac{D(s)}{s_o + s_p D(s)} \tag{16}$$

2) Identification of $D(s)$ and of piezoelectric constant d_p: In order to identify the dynamic part $D(s)$ and the piezoelectric constant d_p, a step voltage with $20V$ of amplitude is applied to the piezocantilever without contact with any objects. A second order model can sufficiently model a lot of system particularly piezocantilevered structures. Using the *output error* method and the matlab software, the model $d_p.D(s)$ was identified and plotted with the experimental result (Fig. 6). This shows the adequacy between both.

$$D(s) = \frac{1}{6.627 \times 10^{-8}s^2 + 5.268 \times 10^{-6}s + 1} \tag{17}$$

and $d_p = 0.6533\mu m/V$.

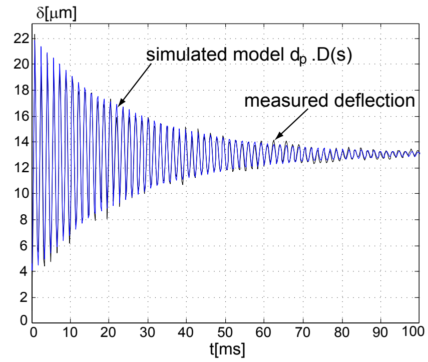

Fig. 6. Step response of the piezocantilever: the measured deflection compared with the simulation of $d_p.D(s)$.

The main objective is the control of the manipulation force for multiple micro-objects having different characteristics. In this work, the experiment will be done with a flexible and a rigid beams with compliances $s_{of} = 6.4237\mu m/mN$ and $s_{or} = 1.7123\mu m/mN$ for the flexible and rigid beams respectively. From this, an interval compliance is derived: $[s_o] = [1.7123, 6.4237]\mu m/mN$. The synthesized controller has to ensure some required performances for any object having a compliance inside this interval.

The interval model describing the transfer voltage-force is given by:

$$[G](s, [s_o]) = d_p \frac{D(s)}{[s_o] + s_p D(s)} \tag{18}$$

After replacing s_p, d_p, $D(s)$ and $[s_o]$ computed previously, we obtain the interval model $[G](s, [a])$:

$$[G](s, [a]) = \frac{1}{[a_2]s^2 + [a_1]s + [a_0]} \tag{19}$$

such as: $[a] = [[a_0], [a_1], [a_2]]$, $[a_0] = [5.682, 12.894]$, $[a_1] = [4.51, 16.92] \times 10^{-6}$ and $[a_2] = [17.369, 65.161] \times 10^{-8}$.

In order to increase the stability margin of the closed-loop system, we propose to extend the intervals of the model (19). When the interval width of the parameters in the model is too large, it is difficult to find a controller that respects both the stability and performances of the closed-loop. After some experiences, we choose to expand the interval width of each parameter of (19) by 10%. 10% represents the maximal value allowed in this application. Finally, the extended parameters of the interval model which will be used to compute the controller are given as follows:

$$\begin{aligned} [a_0] &= [5.3217, 13.2547] \\ [a_1] &= [3.88, 17.55] \times 10^{-6} \\ [a_2] &= [14.97, 67.56] \times 10^{-8} \end{aligned} \tag{20}$$

The interval model $[G](s, [a])$ with the interval parameters given in (20) is valuable for a set of beams having compliances in the range $[so] = [1.7123, 6.4237]\mu m/mN$.

C. Computation of the reference interval model

We want to satisfy the following specifications for the closed-loop:

- no overshoot,
- settling time $15ms \leq tr_{5\%} \leq 80ms$,
- static error $|\varepsilon| \leq 1\%$.

Based on the condition in (12), we choose a second order model such as:

$$[H](s,[K_p],[\tau]) = \frac{[K_p]}{([\tau]s+1)\left(\frac{[\tau]}{10}s+1\right)} \quad (21)$$

(21) can be rewritten as follows:

$$[H](s,[K_p],[\tau]) = \frac{1}{0.1\frac{[\tau]^2}{[K_p]}s^2 + 1.1\frac{[\tau]}{[K_p]}s + \frac{1}{[K_p]}} \quad (22)$$

Where the parameters $[K_p]$ and $[\tau]$ define the specified static error and settling time respectively:

- $[K_p] = 1+\varepsilon = [0.99, 1.01]$,
- $[\tau] = \frac{[tr_{5\%}]}{3} = [5ms, 26.66ms]$.

D. Computation of the interval controller

Using (11), we obtain the following interval controller:

$$[C](s,[a],[K_p],[\tau]) = \frac{[a_2]s^2+[a_1]s+[a_0]}{0.1\frac{[\tau]^2}{[K_p]}s^2 + 1.1\frac{[\tau]}{[K_p]}s + \frac{1}{[K_p]} - 1} \quad (23)$$

After replacing each parameter in (23), we get:

$$[C](s,[c]) = \frac{[c_2]s^2+[c_1]s+[c_0]}{[c_5]s^2+[c_4]s+[c_3]} \quad (24)$$

with: $[c_5] = [0.247, 7.182]\times 10^{-5}$, $[c_4] = [0.544, 2.962]\times 10^{-2}$, $[c_3] = [-0.01, 0.01]$, $[c_2] = [a_2]$, $[c_1] = [a_1]$ and $[c_0] = [a_1]$.

V. Implementation and Experimental Results

The midpoint of each interval parameters of $[C](s)$ (24) has been taken for the implementation of the controller.

$$C_{mid}(s) = \frac{0.4127s^2 + 10.72s + 9.288\times 10^6}{s(25.07s + 1.742\times 10^4)} \quad (25)$$

The closed-loop experiments are performed on the two beams (flexible and rigid). First, a step response analysis is performed. Fig. 7 shows the experimental results when a step reference of $20mN$ is applied. To check that the implemented controller ensures the specifications, the temporal envelope of the wanted interval model $[H](s,[K_p],[\tau])$ is also plotted in the same figure. Especially, we mean by the envelope of the wanted interval model, the step responses of two transfer functions, the first is with the minimum settling time $tr = 15ms$ and maximum static error $\varepsilon = 0.01$, while the second is with the maximum settling time $tr = 80ms$ and minimum static error $\varepsilon = -0.01$. As shown on Fig. 7 the controller has played its role and satisfied the specifications. Indeed, the settling times are: $15ms \leq tr_1 = 36ms \leq 80ms$, $15ms \leq tr_2 = 35ms \leq 80ms$ respectively for the flexible and the rigid beams, the static errors are neglected and belong to the required interval $|\varepsilon| \leq 1\%$.

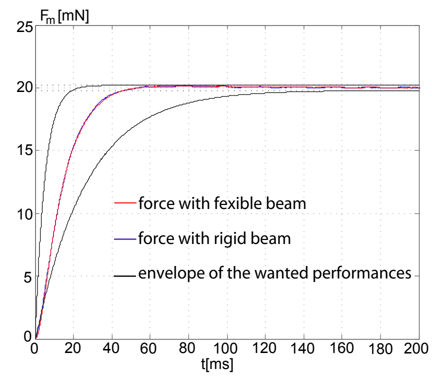

Fig. 7. Step response of the closed-loop: envelope of the wanted interval model compared with the experimental results with $C_{mid}(s)$.

Next, a harmonic analysis is performed. For that, a *sine* reference input of different frequencies is applied. The resulting magnitudes with the two beams, are plotted in Fig. 8. The magnitude envelope of the wanted closed-loop is also plotted in the same graph. It is clear that for any compliance of the object inside the interval $[s_o] = [1.7123, 6.4237]\mu m/mN$, the controller ensures the required performances. As seen on the figure, the different plots show that the wanted interval model includes the experimental results, except in high frequencies because of the neglected high frequency dynamics in the model.

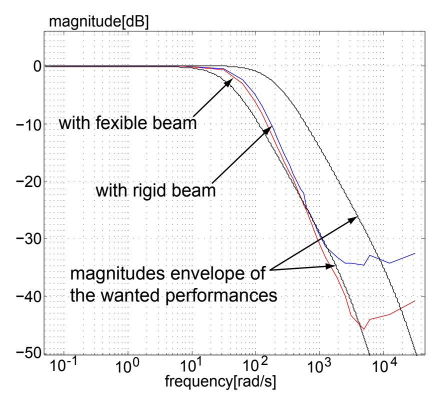

Fig. 8. Magnitudes of the wanted interval model compared with the experimental results with $C_{mid}(s)$.

VI. Performances Analysis

In this part, we prove the choice of the midpoint parameters of the interval controller (see 23 and 25). The idea consists to demonstrate that this midpoint controller ensures the specified performances for any system inside the interval

model, i.e $\forall G(s) \in [G](s,[a])$.
Consider an interval controller $[C](s,[c])$ (see 23):

$$[C](s,[c]) = \frac{[c_2]s^2 + [c_1]s + [c_0]}{[c_5]s^2 + [c_4]s + [c_3]} \quad (26)$$

Based on this controller (26) and on the interval model, we compute the closed-loop:

$$[H_{cl}](s,[a],[c]) = \frac{[C](s,[c]) \cdot [G](s,[a])}{1 + [C](s,[c]) \cdot [G](s,[a])} \quad (27)$$

The objective is to compute the set parameters S_c of the controller that garantee the specifications defined in section. IV-C (22) for $[G](s,[a])$, i.e finding S_c that meet the inclusion $[H_{cl}](s,[a],[c]) \subseteq [H](s)$. Mathematically, this set solution S_c can be written as follows:

$$S_c = \{c \in \mathbf{C} \,|\, [H_{cl}](s,[a],[c]) \subseteq [H](s)\} \quad (28)$$

This problem is known as a set-inclusion problem which can be solved using the SIVIA algorithm (Set Inversion Via Interval Analysis) proposed by [9]. SIVIA algorithm is limited to treat a set-inclusion problem with three parameters (time computation increases exponentially with the number of parameters). According to (26) the controller $[C](s,[c])$ has six parameters. Since the parameters in the numerator are function of those of the interval model, we propose to set $[c_2] = mid([a_2])$, $[c_1] = mid([a_1])$, and $[c_0] = mid([a_0])$. Furthermore, we fix $[c_3] = 0$ in order to cancel the static error. Finally, we work with the remaining parameters $[c_5]$ and $[c_4]$.

After the application of the SIVIA algorithm with an accuracy of $\varepsilon = 10^{-7}$ and an initial box $[c_{50}] \times [c_{40}] = [2 \times 10^{-5}, 4.5 \times 10^{-5}] \times [0.015, 0.018]$, we obtained the subpaving given in the Fig. 9. The dark colored subpaving (S_c) corresponds to the set parameters $[c_5]$ and $[c_4]$ of the controller (26) with $[c_2] = mid([a_2])$, $[c_1] = mid([a_1])$, and $[c_0] = mid([a_0])$, and $c_3 = 0$ that ensure the performances for the interval model. As we can see in the figure, the controller $C_{mid}(s)$ (25) is inside the obtained set solution and is one of the controllers that meet the required specifications.

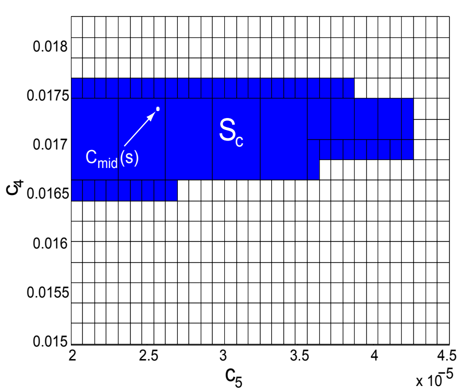

Fig. 9. Set solution of the parameters $[c_5]$ and $[c_4]$ ensuring performances with controller parameters $[c_2] = mid([a_2])$, $[c_1] = mid([a_1])$, $[c_0] = mid([a_0])$, and $c_3 = 0$.

VII. Conclusion

In this paper, a simple method to synthesize robust controllers by combining interval computation and direct synthesis control method has been presented. The proposed approach is based on a given interval model and a wanted interval closed-loop. An interval controller has been derived by applying interval arithmetic. We prove that the midpoint of this interval controller is one of the robust controllers ensuring the performances. The proposed control law was applied to the control of force in piezocantilevers. The experimental results demonstrate the efficiency of the proposed method.

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