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**NEWCASTLE**  
AUSTRALIA

# Measurement and Control for High-Speed Sub-Atomic Positioning in Scanning Probe Microscopes

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**the easy lab**  
electroactive systems and control

*University of Nevada - Reno, Mechanical Engineering Department*



**che easy lab**

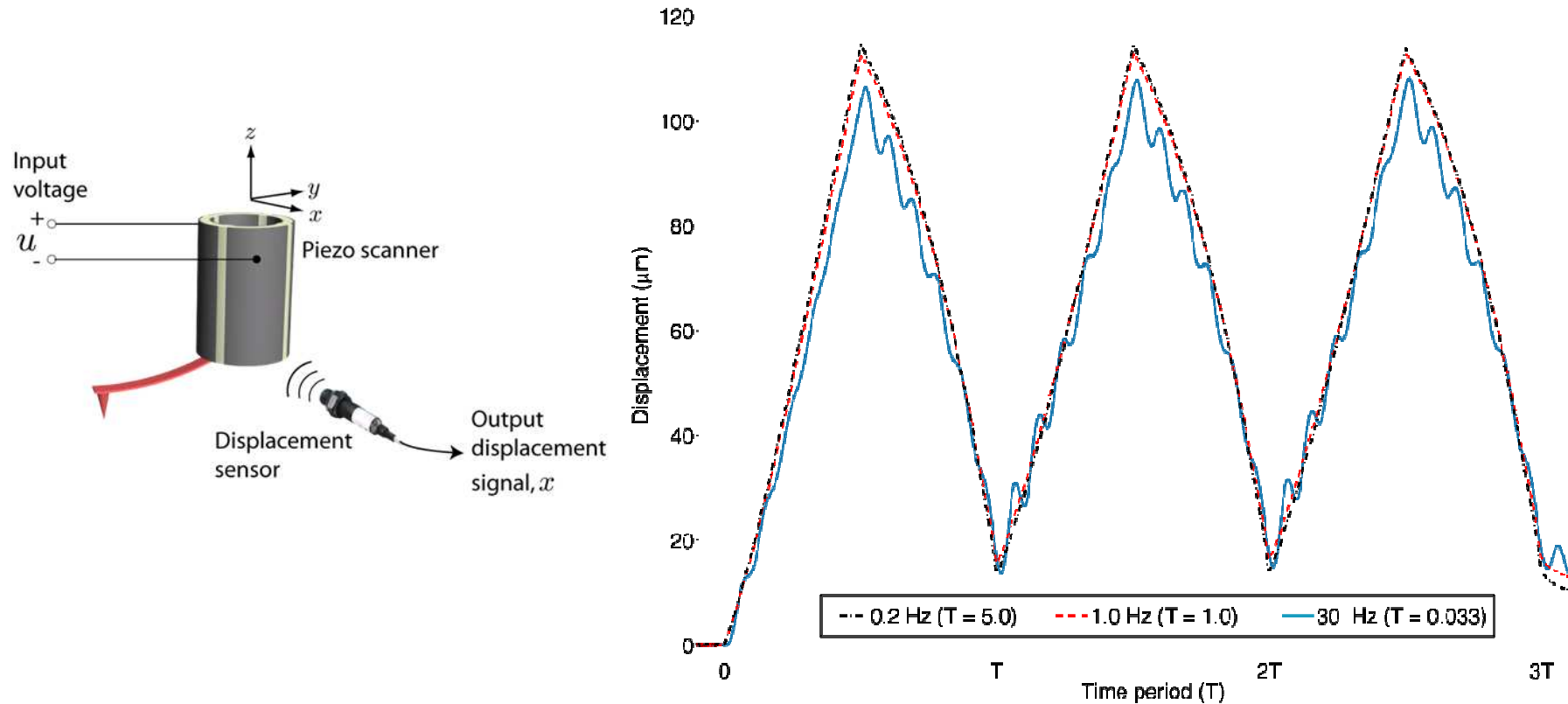
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# Outline

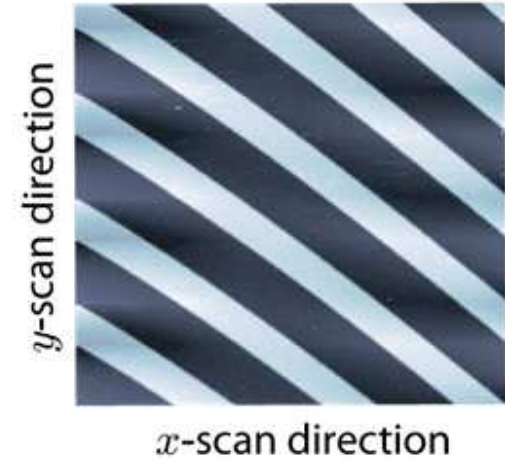
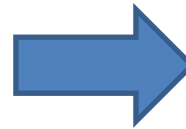
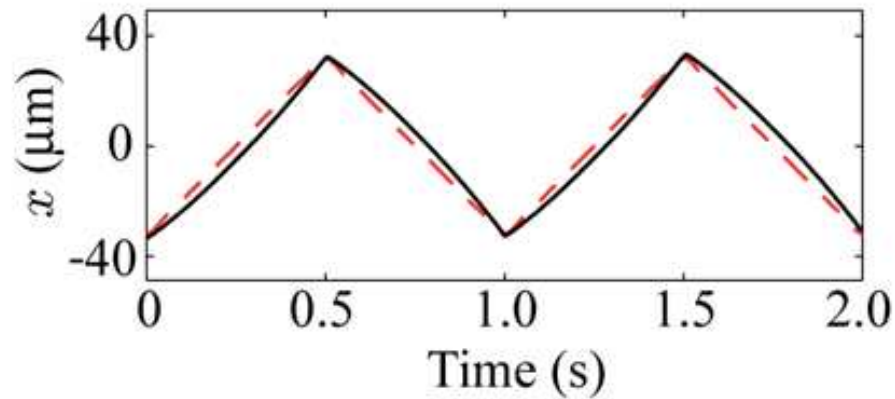
- Challenges and benefits of feedforward control
- System inversion
  - Linear dynamics (creep and vibration)
  - Nonlinearity (hysteresis)
- Iteration-based feedforward
- Conclusions

# Key challenges in SPMs using piezoelectric actuators

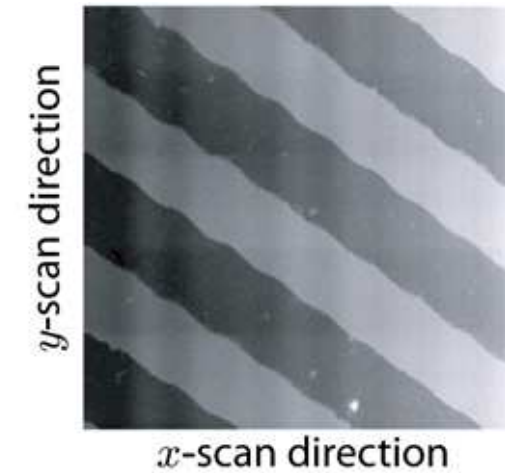
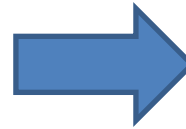
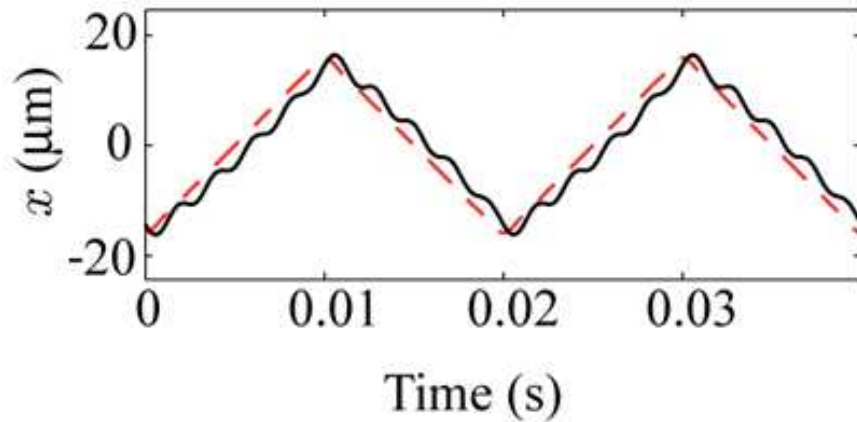


1. Creep: causes drift
2. Hysteresis: causes distortion
3. Vibration: limits bandwidth

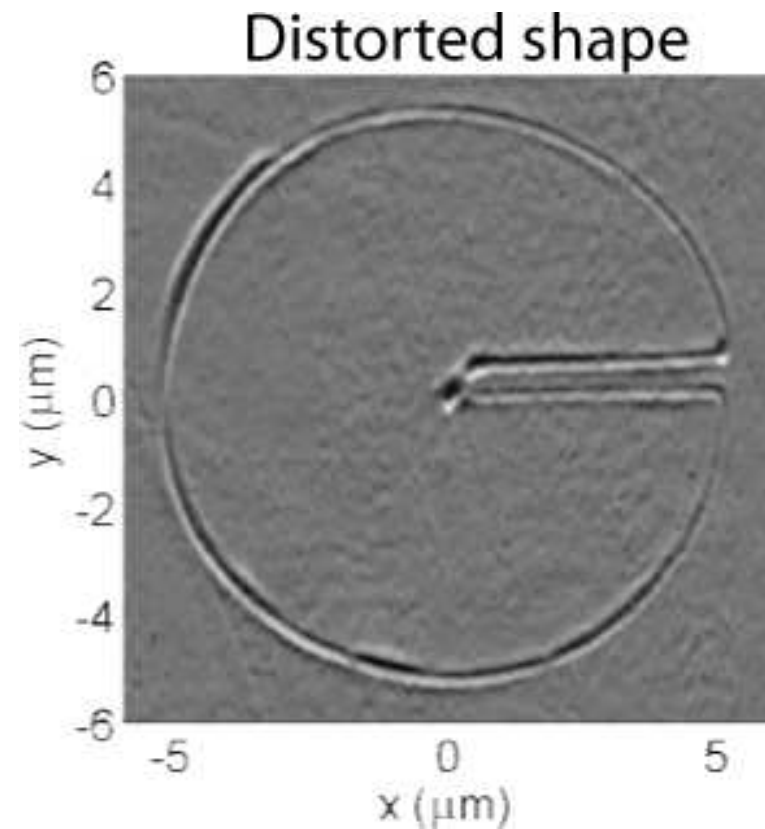
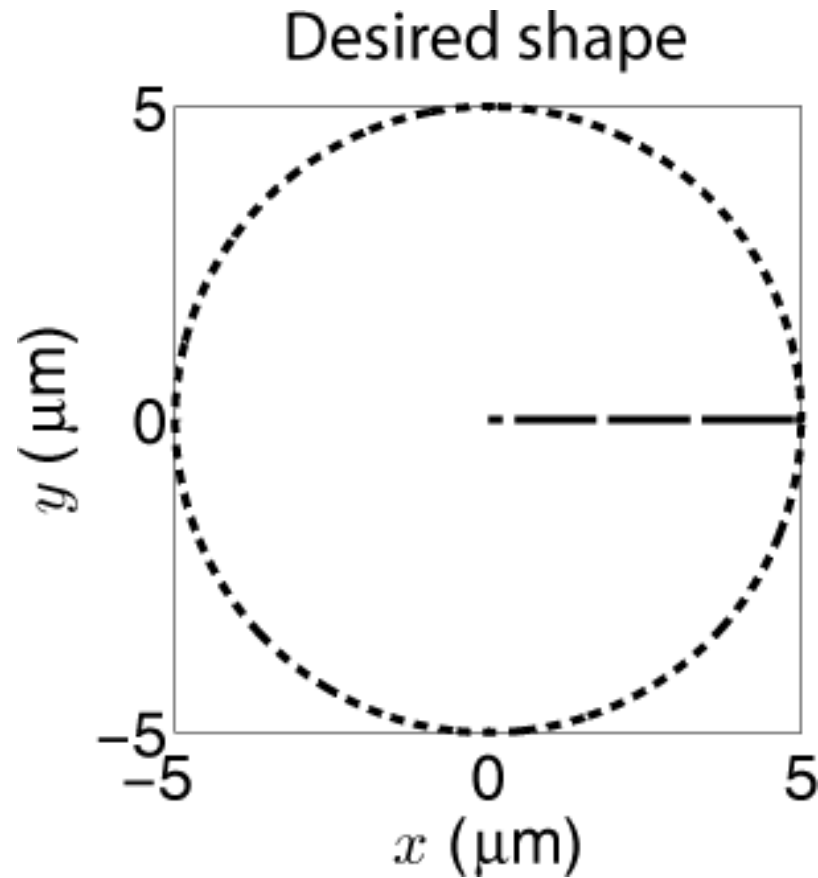
# Positioning errors effect SPM imaging



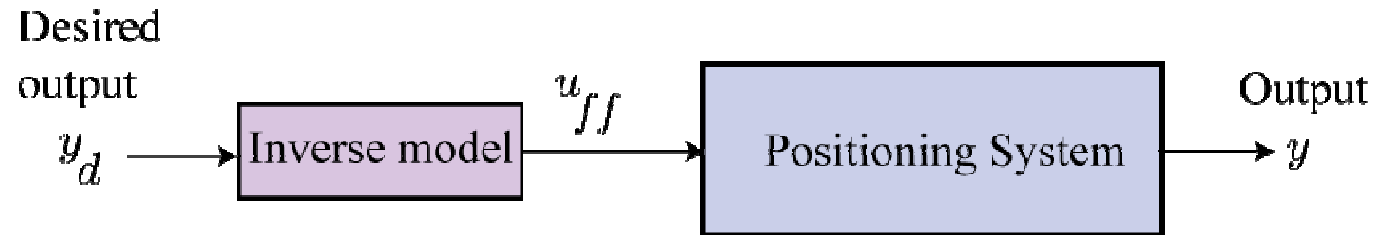
--- Desired — Measured



# Positioning errors effect SPM fabrication



# The feedforward control concept



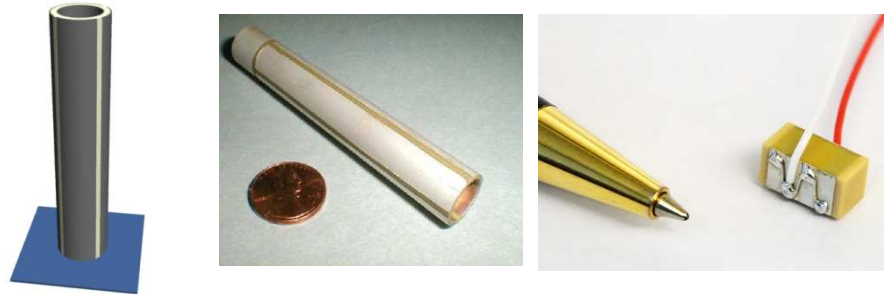
“ Anticipates ”

## Benefits of feedforward control

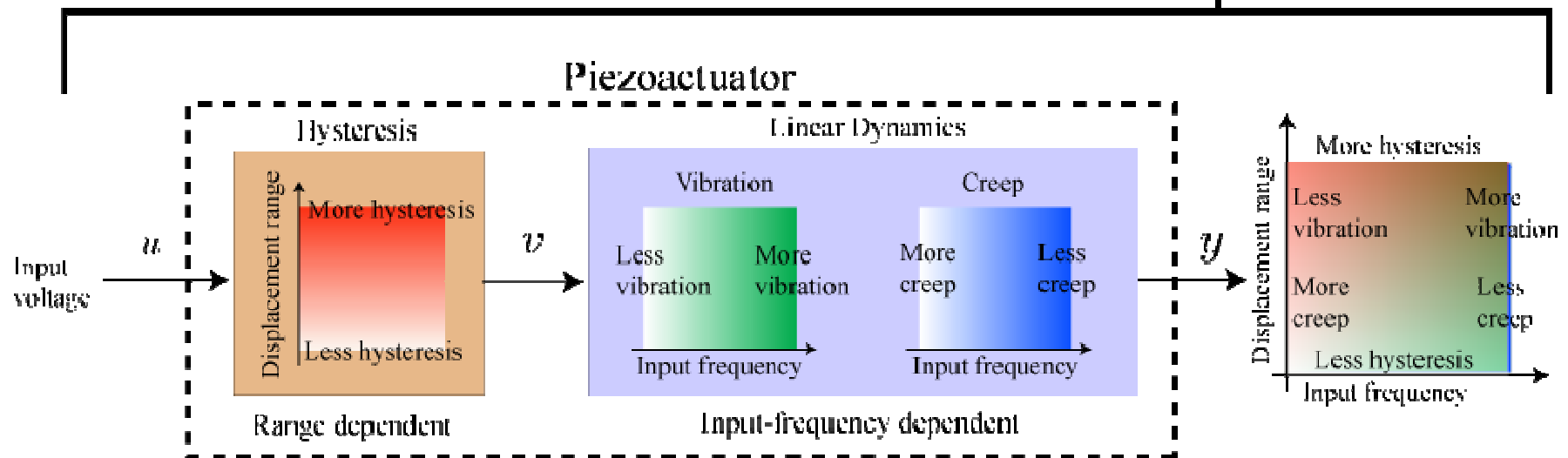
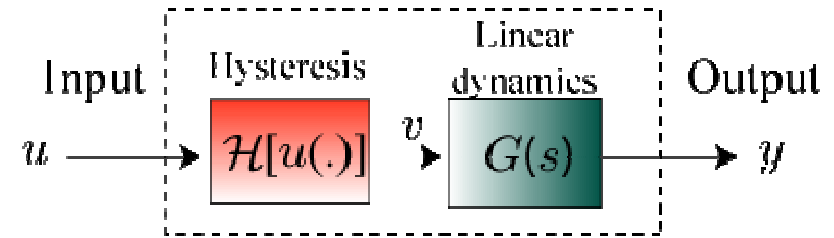
- High-bandwidth positioning (compensates for lag)
- Small tracking error with good models
- Stable
- Cost effective (no sensors for feedback)

# Hammerstein-based model for piezoactuators

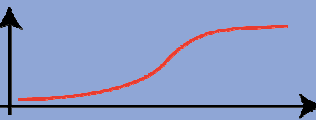
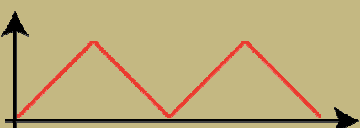
Piezoelectric actuators



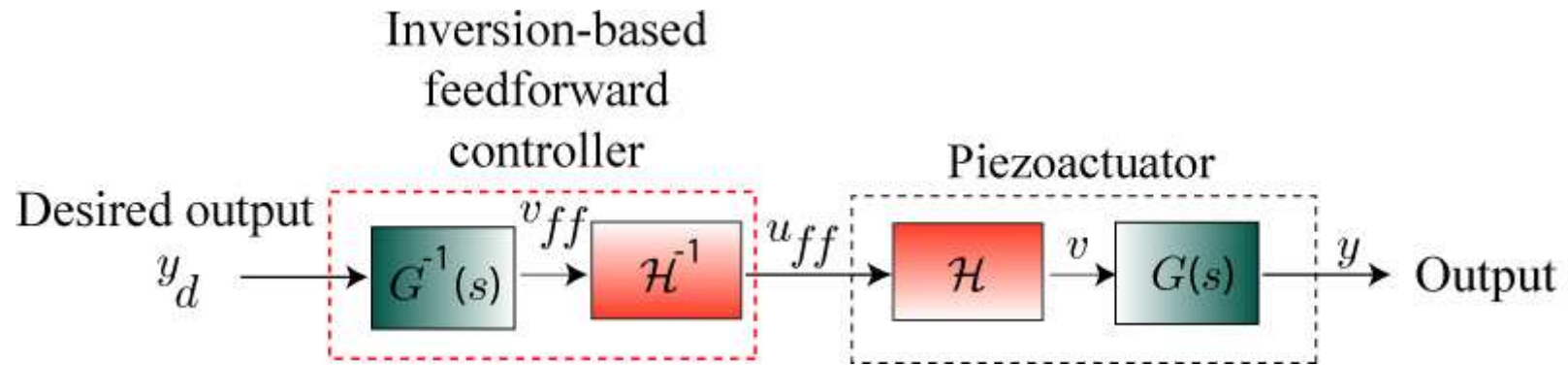
Piezoactuator



# What's the 'best' approach?

Type of SPM operation	
<b>A. Nonrepeating trajectory</b> Example: nanofabrication 	<b>B. Repeating trajectory</b> Example: imaging 
Preferred feedforward-based method	
<b>A1. Model and invert all effects</b> <i>Challenges:</i> Computationally intense; lacks robustness. <i>Advantage:</i> Best performance because all effects are inverted.	<b>B1. Model and invert dynamics; then iterate</b> <i>Challenges:</i> Causality; convergence; nonlinearity of inverse. <i>Advantage:</i> Ability to correct for model uncertainty compared to A1.
<b>A2. Model and invert dynamics; use high-gain feedback or charge control for hysteresis</b> <i>Challenges:</i> Low gain margin; complex circuitry; sensor-induced noise; tracking performance lower than A1. <i>Advantage:</i> Linear inverse vs. nonlinear inverse.	<b>B2. Model and invert dynamics; use high-gain feedback or charge control for hysteresis; then iterate</b> <i>Challenges:</i> Low gain margin; complex circuitry; sensor-induced noise; large input error compared to B1. <i>Advantage:</i> Each iteration does not require a nonlinear inverse.

# A1. Model and invert all effects



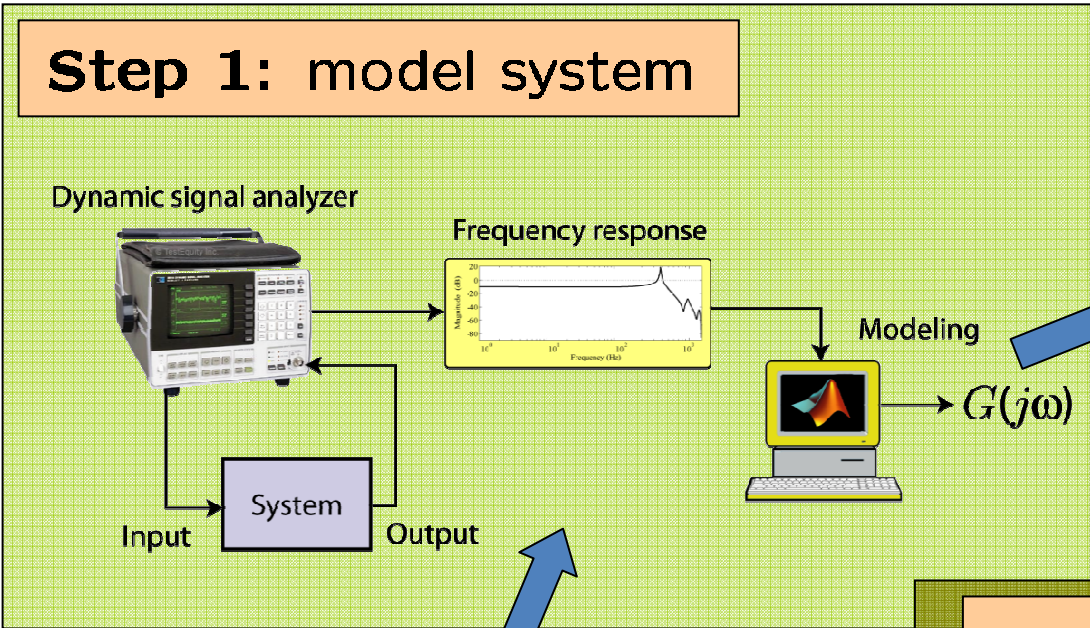
Linear
<ul style="list-style-type: none"><li>• Creep + Vibration</li><li>• Short-range</li><li>• Low-to-high speed</li></ul>

Short-range, low/high-speed

Nonlinear
<ul style="list-style-type: none"><li>• Hysteresis</li><li>• Long-range</li><li>• Low-speed</li></ul>

Long-range, low-speed

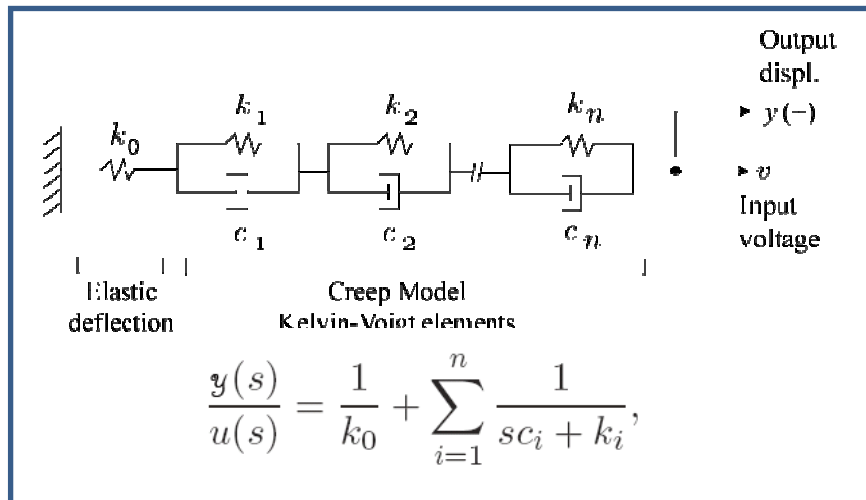
# Short-range, low- and high-speed



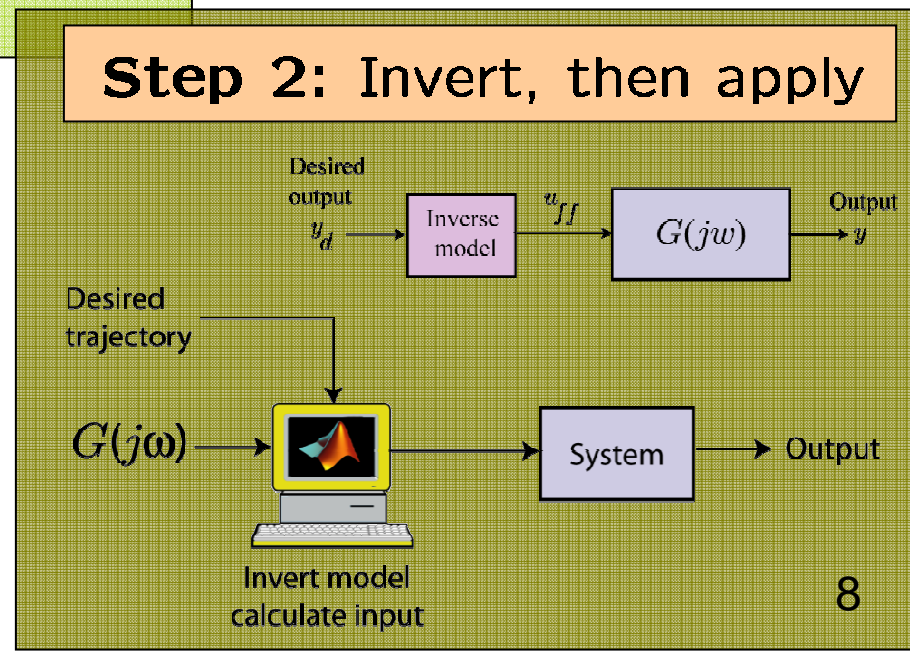
Vibration model

$$G(j\omega) = \frac{\prod^m (j\omega - z_i)}{\prod^n (j\omega - p_k)}; \quad m \leq n$$

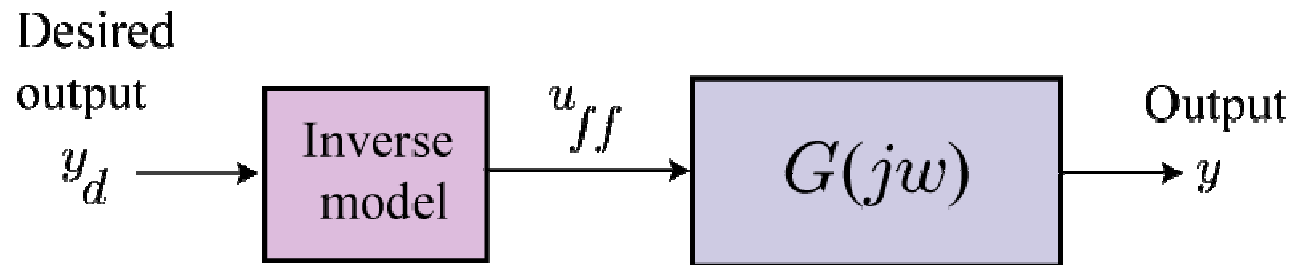
Creep model



**Step 2: Invert, then apply**



# Inversion-based feedforward control: time domain



Consider the (square) linear system:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t)\end{aligned}$$

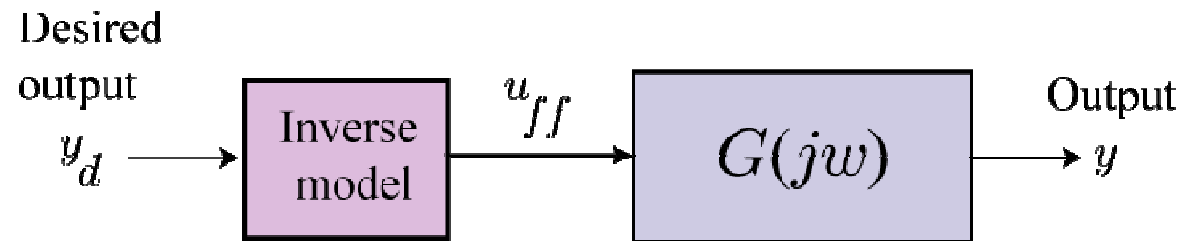
Differentiate the output until the input appears explicitly

$$\begin{aligned}\dot{y}(t) &= C\dot{x}(t) \\ \dot{y}(t) &= CAx(t) + CBu(t); \quad \text{continue until } CA^{r-1}B \neq 0 \\ \vdots &= \vdots\end{aligned}$$

Solve for the feedforward input  $u(t)$ :

$$u_{ff}(t) = (CA^{r-1}B)^{-1} \left[ y_d^{(r)}(t) - CA^r \bar{x}(t) \right] \quad (r = \text{relative degree})$$

# Inversion-based feedforward: frequency domain



$$u_{ff}(j\omega) = G^{-1}(j\omega)y_d(j\omega) \quad (\text{Exact Inversion})$$

**But inputs may be excessively large!**

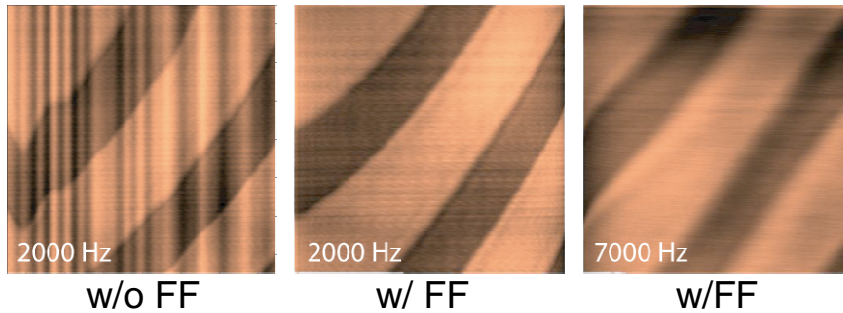
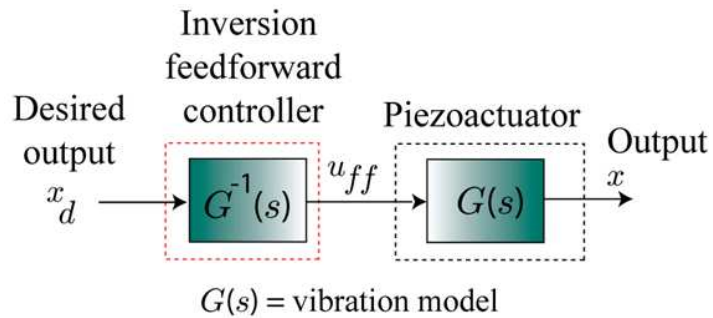
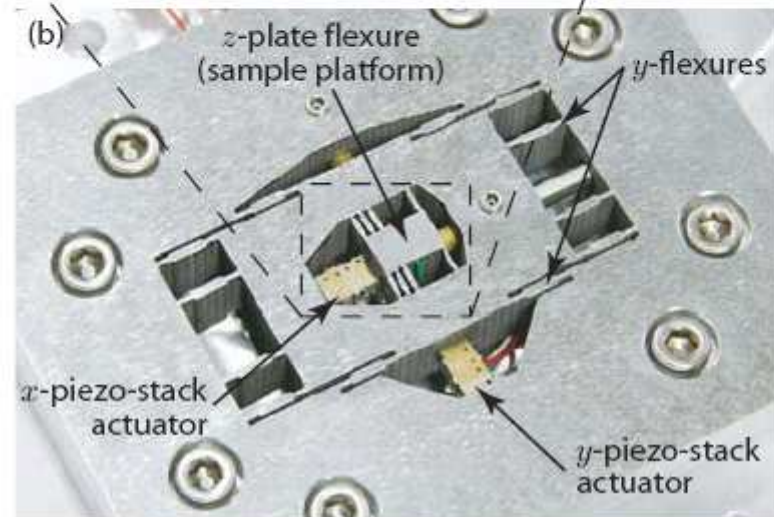
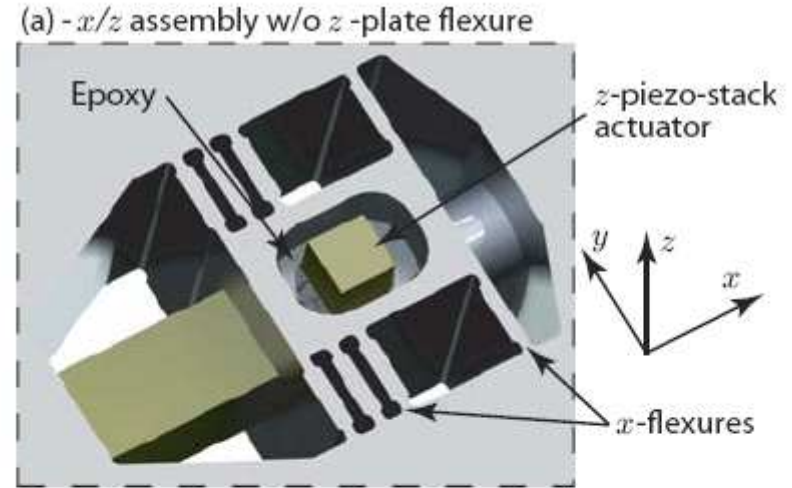
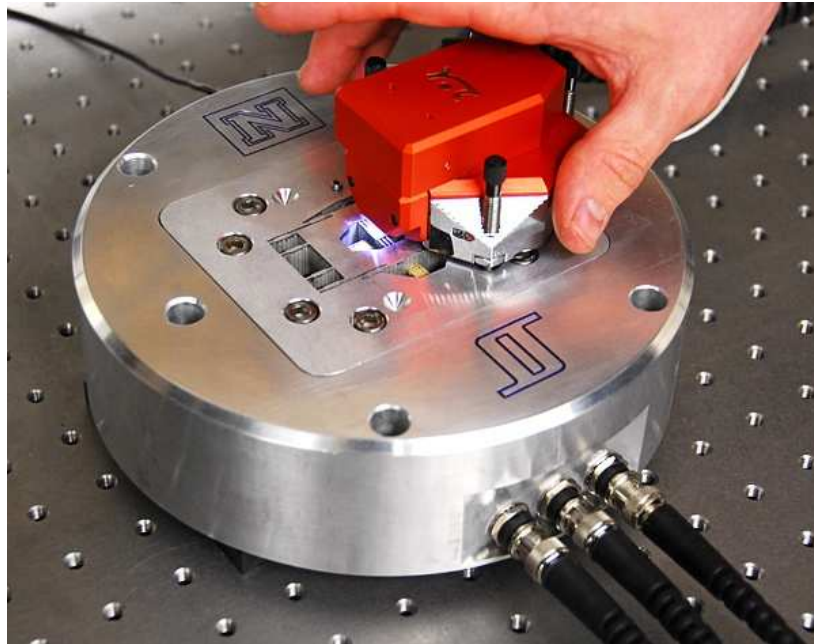
Find input subject to the cost:

$$J(u) = \int_{-\infty}^{\infty} \left( u^*(j\omega)R(j\omega)u(j\omega) + [y(j\omega) - y_d(j\omega)]^* Q(j\omega) [y(j\omega) - y_d(j\omega)] \right) d\omega$$

$$u_{opt}(j\omega) = \left[ \frac{G^*(j\omega)Q(j\omega)}{R(j\omega) + G^*(j\omega)Q(j\omega)G(j\omega)} \right] y_d(j\omega) \quad (\text{Optimal})^\dagger$$

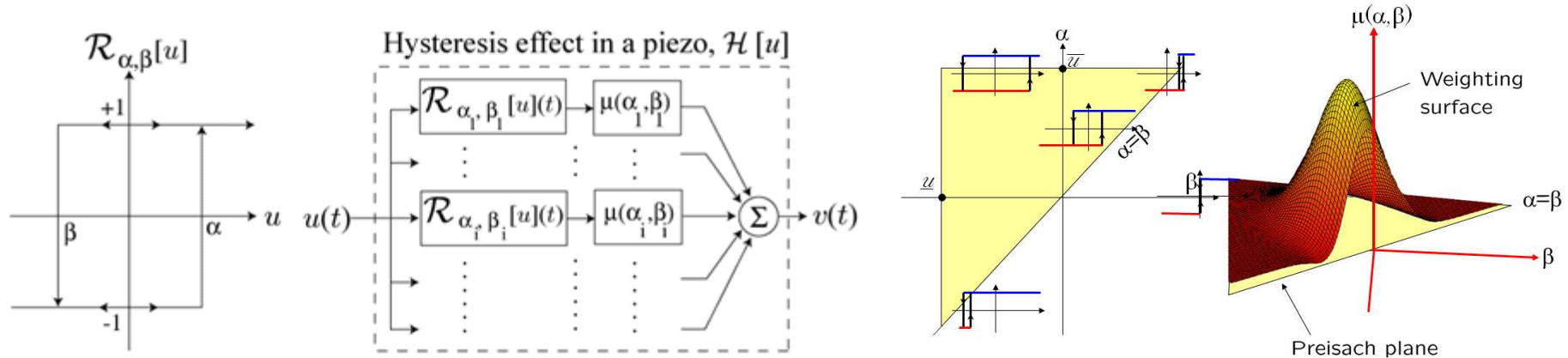
\* e.g., Leang and Devasia, 2007.

# High-speed AFM imaging example



Range: 10 x 10 x 2  $\mu\text{m}$   
 Resonances: 25 kHz (x),  
 6 kHz (y),  
 >80 kHz (z)  
 AFM imaging rate: >70 fps (100x100 pixels)

# The Preisach hysteresis model



- Assume material consists of an infinite collection of basic relays,

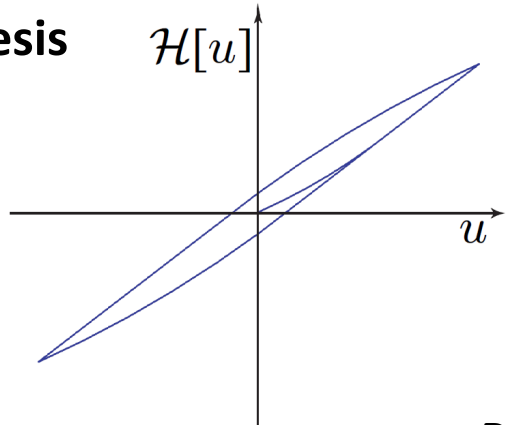
$$(\text{Basic relay}) \mathcal{R}_{\alpha,\beta}[u](t) = \begin{cases} +1 & u(t) > \alpha, \\ -1 & u(t) < \beta, \\ \text{unchanged} & \beta \leq u(t) \leq \alpha, \end{cases}$$

- Assume output is a weighted sum of such relays, i.e.,

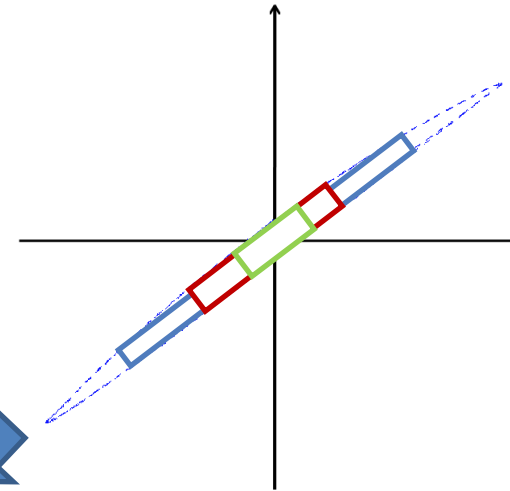
$$v(t) = \mathcal{H}[u](t) = \iint_{\alpha \geq \beta} \mu(\alpha, \beta) \mathcal{R}_{\alpha,\beta}[u](t) d\alpha d\beta,$$

# Prandtl-Ishlinskii hysteresis model

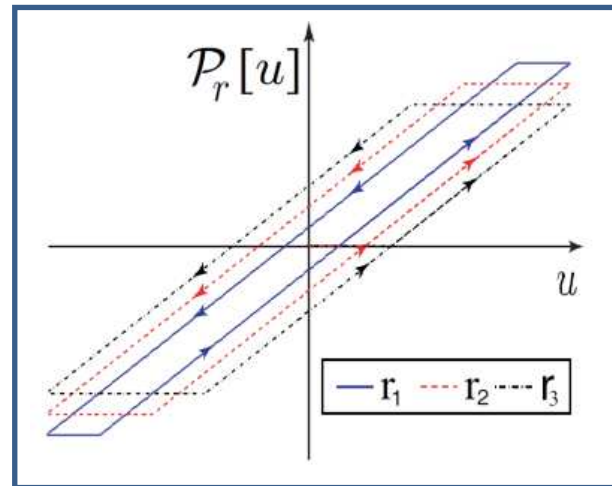
Hysteresis curve:



Sum of basic relays



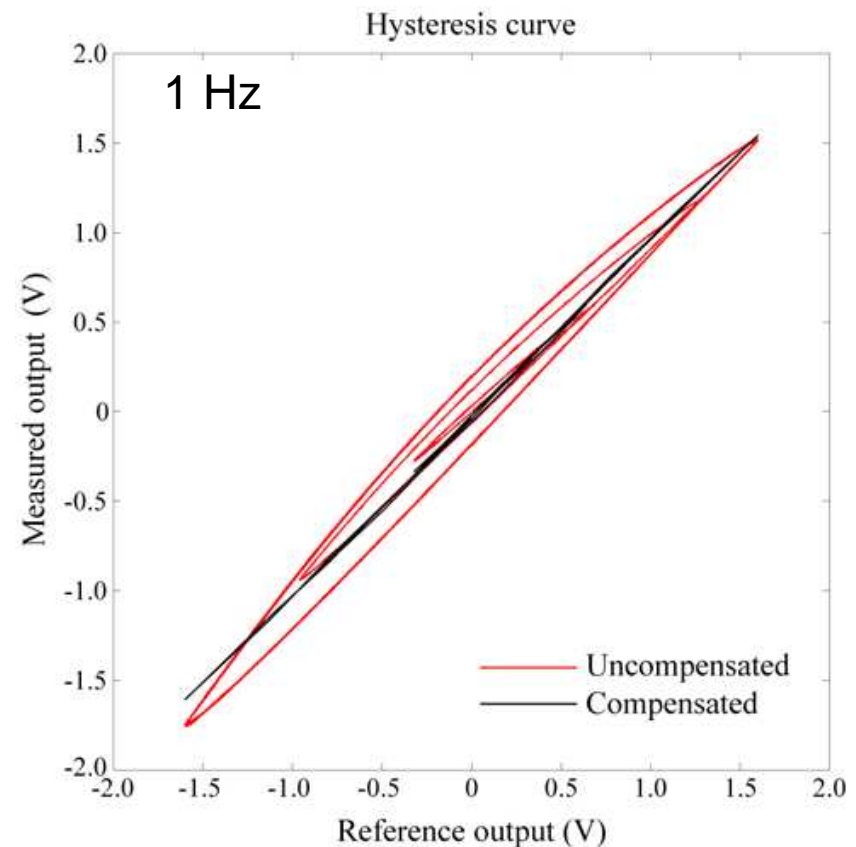
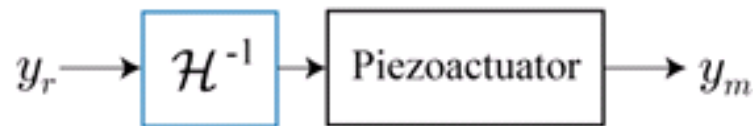
Play operator



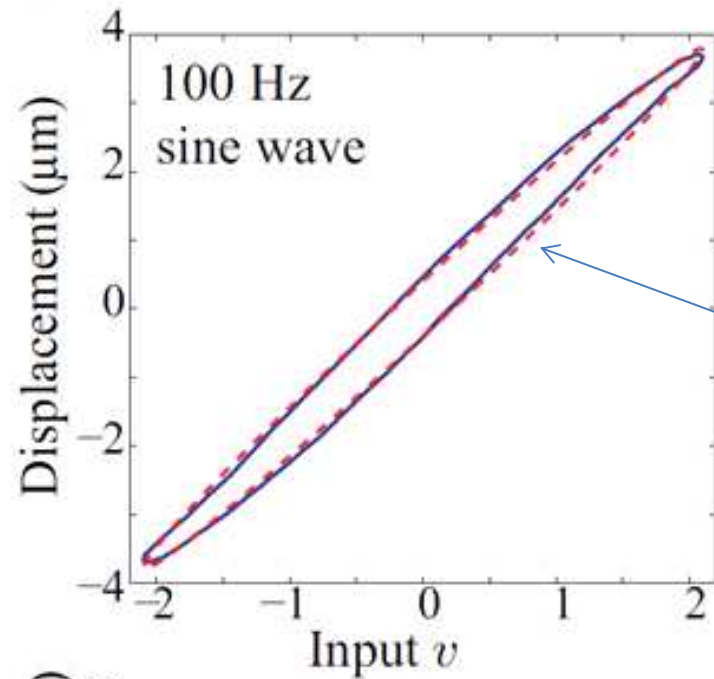
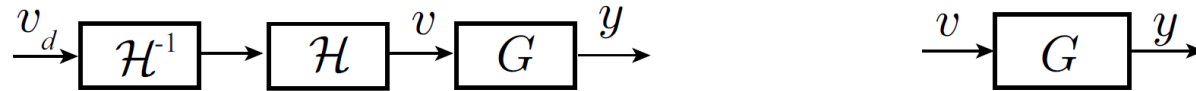
$$v(t) = \mathcal{H}[u](t) \triangleq kf(t) + \int_0^R d(r)\mathcal{P}_r[u](t)dr$$

# Advantages of relay-based hysteresis models

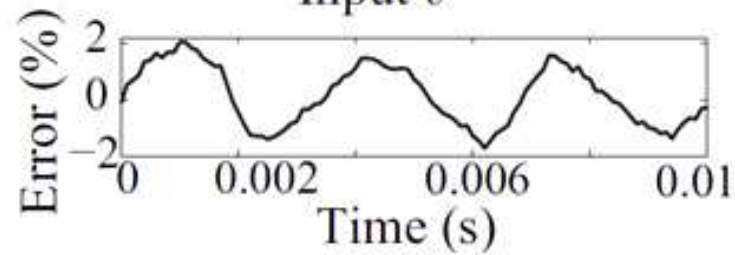
- Inverse model can be obtained from measured input/output data
- Can be use for real-time feedforward control



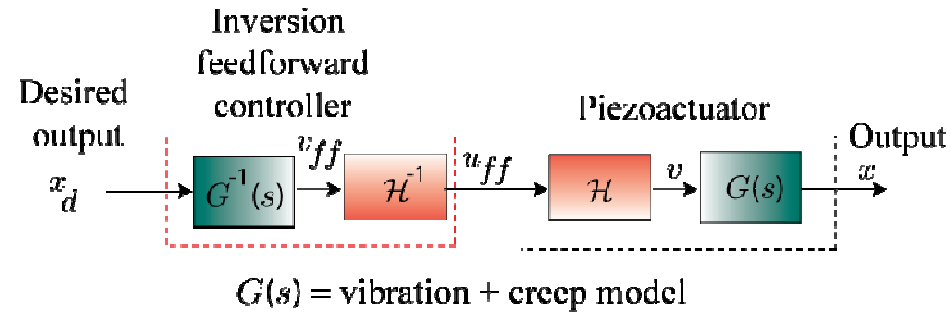
— Measured response with  $\mathcal{H}^{-1}$       - - - Simulated output of  $G$



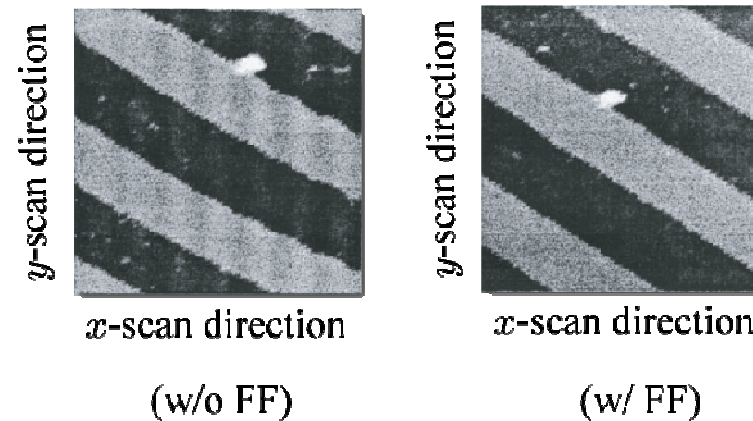
Loop due to phase shift



# Creep, hysteresis, and vibration compensation

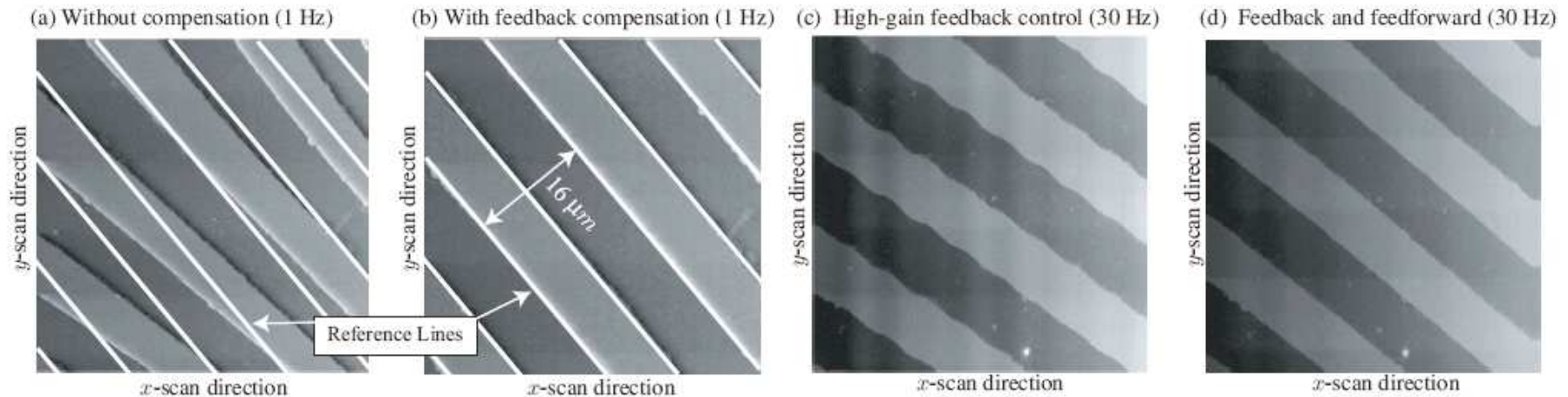
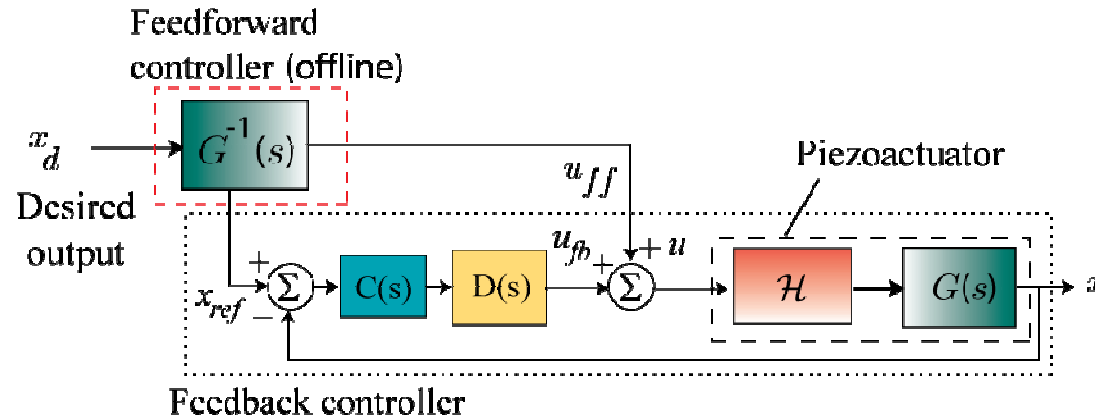


(30 Hz scanning)



Advantage: offers best performance with good models

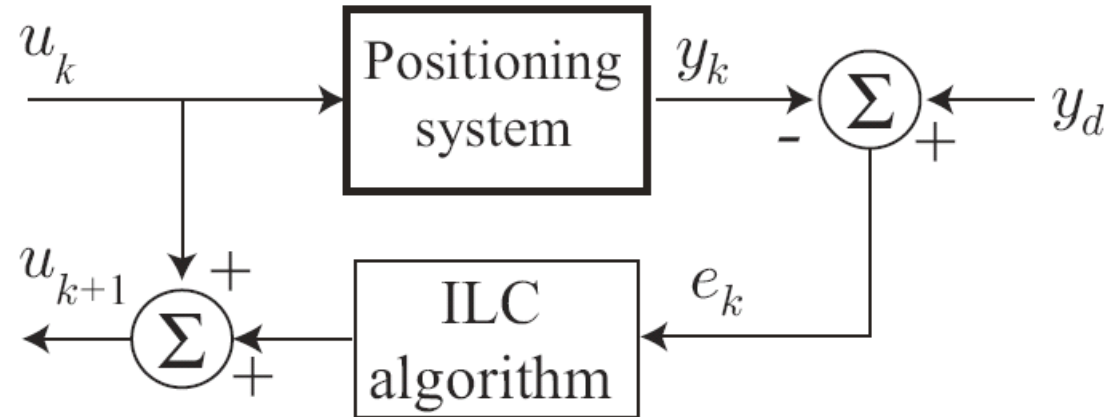
## A2. Model and invert $G(j\omega)$ , use feedback for $H$



Advantage: avoids modeling nonlinearity

# B1. What if you don't know the model?

Iteratively find the feedforward input



$$u_{k+1}(t) = u_k(t) + f(e_k(t)), \quad \text{for all } t \in I \triangleq [t_0, T]$$

## Advantages

- Does not require an accurate model
- Provides high-precision tracking
- Can be automated, but requires a sensor
- Stable

# Types of update laws: time-domain

1. System is linear<sup>1</sup>

$$u_{k+1}(t) = u_k(t) + \rho[y_d^{(r)}(t) - y_k^{(r)}(t)]$$

$$\rho \text{ satisfies } \|I - CA^{r-1}B\rho\|_\infty < 1$$

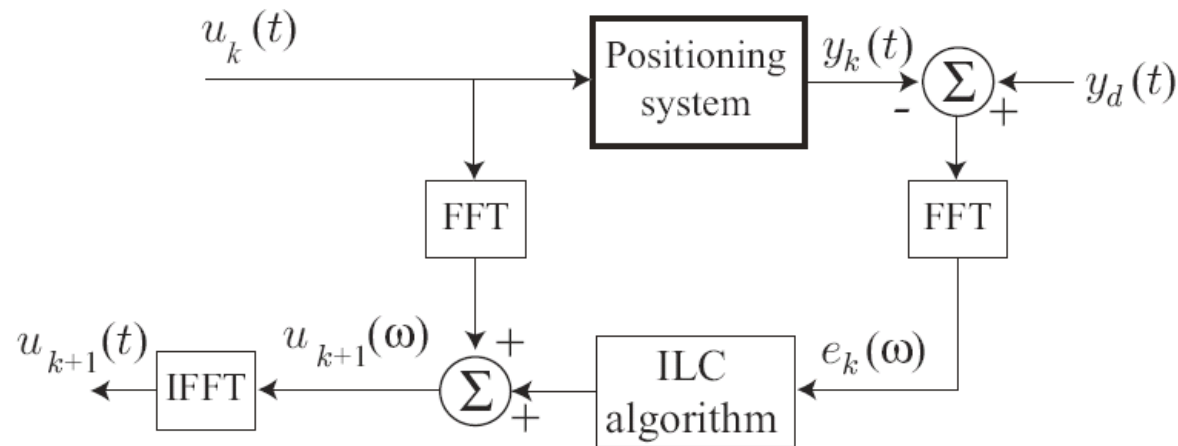
2. System is hysteretic<sup>2</sup>

$$u_{k+1}(t) = u_k(t) + \rho(v_d(t) - v_k(t))$$

$$0 < \rho \leq 1/\eta_2.$$

<sup>1</sup>Arimoto et. al., 1983; <sup>2</sup>Leang and Devasia, 2006

# Frequency domain\*



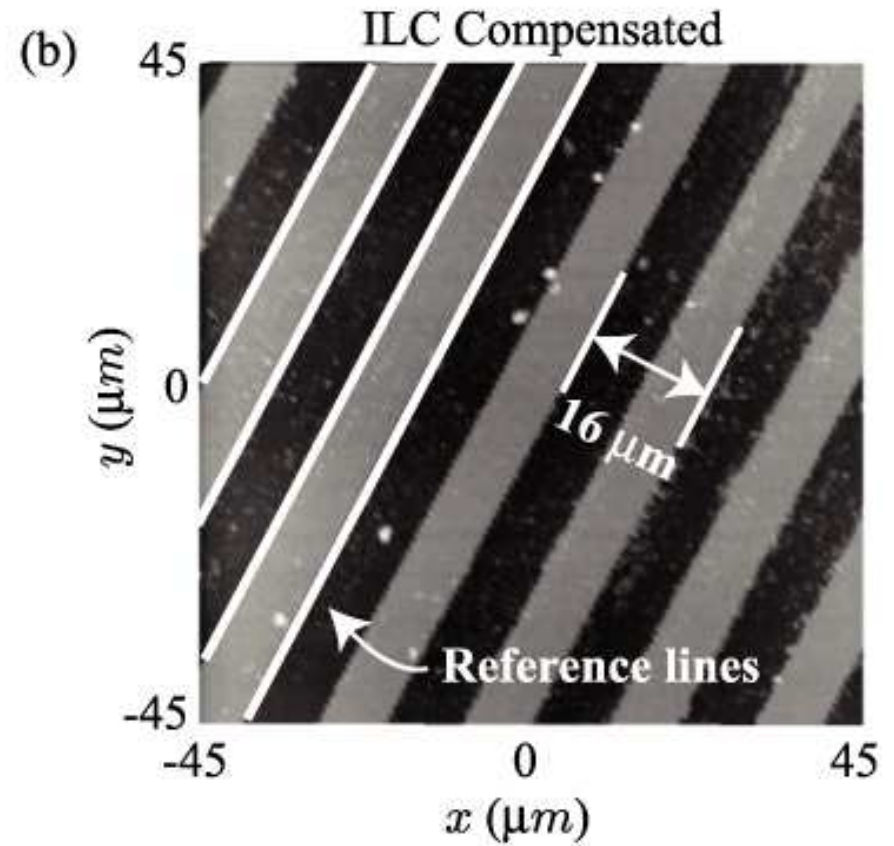
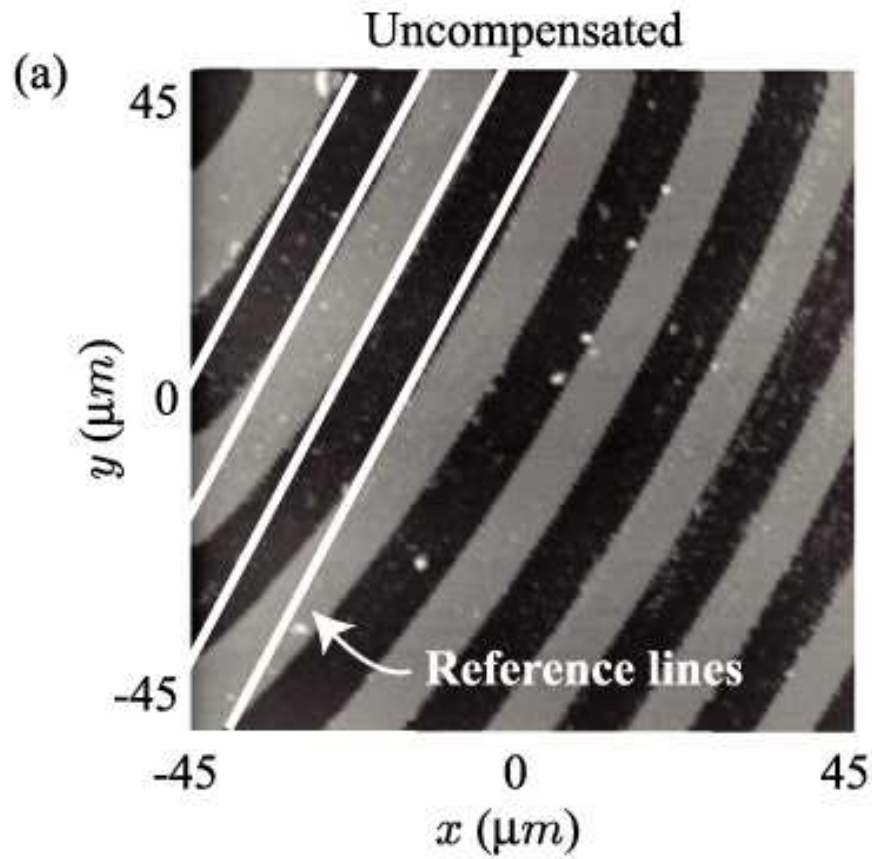
$$u_{k+1}(\omega) = u_k(\omega) + \rho(\omega)\hat{G}^{-1}(\omega)[y_d(\omega) - y_k(\omega)]$$

$$0 < \rho(\omega) < \frac{2 \cos(\Delta(\omega))}{A(\omega)}, \text{ for } \cos(\Delta(\omega)) > 0;$$

$$\rho(\omega) < \frac{2 \cos(\Delta(\omega))}{A(\omega)} < 0, \text{ for } \cos(\Delta(\omega)) < 0;$$

\* Wu et. al., 2007

# AFM imaging example



1 Hz scanning; 100 iterations

# Conclusions

- Feedforward control anticipates for deficit performance
- Feedforward control provides high-bandwidth, high-precision positioning (compensates for lag) with good models
- Feedforward-controlled system is stable
- Feedforward control does not require sensors