Classical Prandtl-Ishlinskii modeling and inverse multiplicative structure to compensate hysteresis in piezoactuators

Micky RAKOTONDRABE, *Member, IEEE*

Abstract— **This paper presents a new approach to compensate the static hysteresis in smart material based actuators that is modeled by the Prandtl-Ishlinskii approach. The proposed approach allows a simplicity and ease of implementation. Furthermore, as soon as the direct model is identified and obtained, the compensator is directly derived. The experimental results on piezoactuators show its efficiency and prove its interest for the precise control of microactuators without the use of sensors. In particular, we experimentally show that the hysteresis of the studied actuator which was initially 23% was reduced to less than 2.5% for the considered working frequency.**

I. INTRODUCTION

Piezoelectric ceramics (piezoceramics) are very prized in the design of microrobots, micro/nanopositioning devices and systems at the micro/nano scale in general. They have been successfully used to develop stepper microrobots [1][2], Atomic Force Microscopes (AFM) [3] and continuous microactuators such as piezocantilevers and microgrippers [4][5]. This recognition is mainly thanks to the high resolution (at the nanometre level), the high bandwidth (more than 1kHz) and the relatively high force density that they offer. However, when the applied electrical field is large, piezoceramics exhibit an important hysteresis nonlinearity which strongly limits the accuracy of the developed actuators.

Three approaches exist to control the hysteresis and to improve the general performance of piezoelectric actuators (piezoactuators): feedback control, charge control, and feedforward voltage control. In feedback control, both classical (PID, ...) and advanced control laws (H_∞, passivity,...) have been successfully used [6][7]. Its main advantages are the possibility to reject external disturbance effects and to account for the model uncertainties. However, the use of closed loop control techniques at the micro/nano scale is strongly limited by the difficulty to integrate sensors. Sensors which are precise and fast enough are bulky (interferometers, triangulation optical sensors, camera-microscopes measurement systems, etc.) or difficult to fabricate. In charge control, an adapted electrical circuit is used to provide the input charge applied to the piezoactuators [8][9][10]. Finally, in feedforward voltage control, the hysteresis is precisely mod-

FEMTO-st Institute,
UMR CNRS-6174 / UFC / ENSMM / UTBM
Automatic Control and Micro-Mechatronic Systems department (AS2M department)
25000 Besançon - France
`mrakoton@femto-st.fr`

eled and a kind of inverse model is put in cascade with the process resulting in an overall linearized system. The main advantage of the two latter approaches is the shunning of external sensors making the controlled system packageable and fabricated with low cost. In an automatic point of view, feedforward voltage control is particularly appreciated because this approach allows stability, performance analysis and controllers synthesis.

For piezoactuators, there exist several approaches of hysteresis compensation based on voltage control: the Bouc-Wen [11], the polynomial [12], the lookup tables [13], the Preisach [14][15] and the classical Prandtl-Ishlinskii approaches [16][17][18]. The Prandtl-Ishlinskii approach is particularly appreciated for its simplicity, ease of implementation and accuracy. It is based on the sum of many elementary hysteresis backlash operators. The accuracy of the model increases with the number of these operators. To compute the corresponding hysteresis compensator, the least-squared error optimization has been used [17]. However the computation time greatly increases according to the number of operators which makes this method only practical for low number of backlash operators. In this paper, we propose another compensation approach for hysteresis modelled by the Prandtl-Ishlinskii. Based on the inverse multiplicative structure, the proposed approach does not need any computation of the compensator. Indeed, as soon as the model is identified, the compensator is derived without extra-calculation. Therefore, independant from the number of the backlashes (and whatever the required accuracy), there is no cost for the compensator computation.

II. THE CLASSICAL PRANDTL-ISHLINSKII MODELING AND IDENTIFICATION

The classical Prandtl-Ishlinskii (PI) model is based on the backlash operator.

A. The backlash operator

Definition 2.1: A backlash operator, also called play-operator (see Fig. 1), is defined by the following equations:
$$\begin{cases} y(t) = \max\left\{u(t) - r, \min\left\{u(t) + r, y(t - T)\right\}\right\} \\ y(0) = y_0 \end{cases}$$

where $u(t)$ is the input control, $y(t)$ is the output displacement, r is the threshold of the backlash and T is the refresh time.
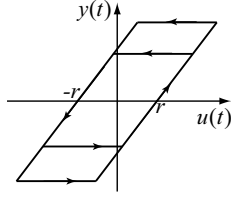
Fig. 1. A backlash operator with a slope unity.

B. The classical PI model

Definition 2.2: A classical PI hysteresis model is defined as the sum of several backlashes each one having a threshold r_i and a slope (weighting) w_i [19]:

$$\begin{cases} y(t) = \sum_{i=1}^{n} w_i \cdot \max\left\{ u(t) - r_i, \min\left\{ u(t) + r_i, y_{ei}(t - T) \right\} \right\} \\ y(0) = y_0 \end{cases}$$

where n is the number of operators and y_{ei} the ith elementary output (output of the ith backlash). Fig. 2 gives the block diagram showing the principle of the classical PI hysteresis modeling.
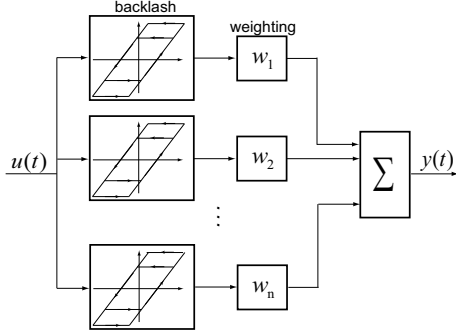

Fig. 2. Diagram showing the principle of the classical PI modeling.

C. Parameters identification

Following the procedure in [16], the identification of the parameters r_i and w_i is performed by applying a sine or a triangular input voltage $u(t)$ with an amplitude u_A to the process. This amplitude corresponds to the maximal output of y that is expected for the applications. The curve in the (u, y)-plane - which has a hysteresis shape - should be afterwards shifted so that it is in the positive section of the plane. Fig. 3 shows an example of a (shifted) hysteresis curve approximated by three backlashes. In Fig. 3, $bw_i = 2 \cdot r_i$ is the bandwidth. From the figure, the kth output can be formulated as follows:

$$y_k = \sum_{i=1}^{k} (bw_{k+1} - bw_i) \cdot w_i \qquad (1)$$

From the previous equation, a tensorial formulation can be obtained:

$$\{y\} = [A] \cdot \{w\} \qquad (2)$$

where $[A]$ is a triangular matrix constructed from the different bandwidth values.
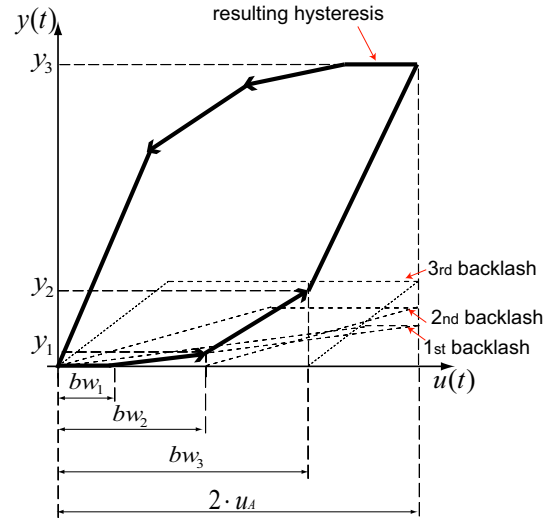

Fig. 3. Example of (shifted) hysteresis obtained with three elementary backlashes.

The identification procedure is therefore as follows [16].

- Apply at least half a period of sine voltage $u(t)$ to the piezoactuator. The amplitude of the corresponding output $y(t)$ should cover the end use range.
- If the obtained hysteresis curve is not in the positive section of the (u, y)-plane, shift the curve.
- Define the number n of the backlashes.
- Split the input u domain into $n + 1$ uniform or non-uniform partitions. For example, Fig. 3 depicts four partitions and presents an approximation of hysteresis with three backlashes. The bandwidth bw_i and the output vector $\{y\}$ are easily obtained according to Fig. 3.
- Construct the matrix $[A]$ from the bandwidth bw_i by using (equ 1) and (equ 2),
- Finally, compute the parameter $\{w\}$ using the following formula:

$$\{w\} = [A]^{-1} \cdot \{y\} \qquad (3)$$

Remark 2.1: The classical PI hysteresis model (see Def. 2.2) is a static model. It is used to model hysteresis of processes working at low frequency. At high frequency, the classical PI hysteresis model is often combined with a linear dynamics to maintain the initial accuracy [16]. Since the compensation of this dynamics part is independant from the compensation of the static hysteresis and is available in several approaches [14][16], this paper only focuses on the static hysteresis.

Remark 2.2: The refresh time T should be low relative to the time characteristics of the used input signals such as the period of $u(t)$. Indeed, if T is high, the backlash defined in Def. 2.1 (and in Fig. 1) is distorted and the accuracy of the PI model in Def. 2.2 is decreased. The choice of T can start with the Shannon Theorem that can be further refined if necessary. For example, from the expected working frequency f, the sampling frequency f_s should satisfy $f_s \geqslant f$.

III. A NEW COMPENSATION APPROACH FOR THE PI HYSTERESIS MODELING

In this section, we propose a new compensation method for the classical PI hysteresis model previously presented. The advantage of the proposed method is that as soon as the model is identified, the compensator is directly derived without additional calculation. For that, we need to rewrite the PI model.

A. General principle

Definition 3.1: The (feedforward) compensation of piezoelectric materials hysteresis consists in putting in cascade with the hysteretic system a compensator (see Fig. 4) such that one obtains a linear input-ouptut (y_r, y) with a unity gain between the reference input y_r and the output y [20]: $\frac{\partial y}{\partial y_r} = 1$

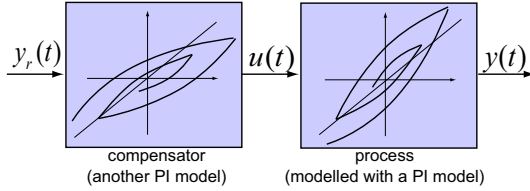*Remark 3.1:* expression $\frac{\partial y}{\partial y_r} = 1$ in Def. 3.1 is similar to $y = y_r$.

Fig. 4.   Compensation of a hysteresis.

B. Rewriting the model

First we shall rewrite the hysteresis model already defined in Def. 2.2. For that, we need to give a property of the backlash operator.

Property 3.1: Reconsider the backlash operator in Def. 2.1. We have: $r = 0 \Leftrightarrow y(t) = u(t)$

So we have the following consequence which is an alternative expression of Def. 2.2.

Consequence 3.1: A classical PI hysteresis model can be expressed as follows:

$$\begin{cases} y(t) = -u(t) \\ + \sum_{i=0}^{n} w_i \cdot \max\left\{u(t) - r_i, \min\left\{u(t) + r_i, y_{ei}(t-T)\right\}\right\} \\ y(0) = y_0 \end{cases}$$

where r_i and w_i (for $i = 1 \cdots n$) are known according to the above identification procedure. For $i = 0$, we have: $r_0 = 0$ and $w_0 = 1$.

Proof: We rewrite the first equation in Def. 2.2 as follows: $y(t) = u(t) - u(t)$

$$+ \sum_{i=1}^{n} w_i \cdot \max\left\{u(t) - r_i, \min\left\{u(t) + r_i, y_{ei}(t-T)\right\}\right\}$$

According to Property 3.1, $u(t)$ can be expressed using the backlash operator by using a threshold $r_0 = 0$. Multiplying the result by a weighting $w_0 = 1$, we obtain: $u(t) = w_0 \cdot \max\left\{u(t) - r_0, \min\left\{u(t) + r_0, y_{e0}(t-T)\right\}\right\}$ Using the two previous equations, we derive Consequence. 3.1. ∎

C. A new compensator for the hysteresis

First, we give a consequence of Remark 2.1 and Remark 2.2 that will be used further.

Consequence 3.2: Define a compensator with input $y_r(t)$ and output $u(t)$. From Remark 2.1 ($\frac{du(t)}{dt}$ is low) and Remark 2.2 (T is very low), we have: $\left| \frac{\partial u(t)}{\partial y_r(t)} - \frac{\partial u(t-T)}{\partial y_r(t)} \right| \to 0$ where $\frac{\partial u(t)}{\partial y_r(t)}$ is the slope of the compensator map $(y_r(t), u(t))$.

Proof: Since $\frac{du(t)}{dt}$ and T are both low, we have $\frac{du(t-T)}{dt}$ also low. Thus, we derive $\left| \frac{du(t)}{dt} - \frac{du(t-T)}{dt} \right| \to 0$ The latter expression can be rewritten as follows: $\left| \frac{\partial u(t)}{\partial y_r(t)} \cdot \frac{dy_r(t)}{dt} - \frac{\partial u(t-T)}{\partial y_r} \cdot \frac{dy_r(t)}{dt} \right| \to 0$ which yields: $\left| \frac{\partial u(t)}{\partial y_r(t)} - \frac{\partial u(t-T)}{\partial y_r} \right| \cdot \left| \frac{dy_r(t)}{dt} \right| \to 0$ For any continuous and differentiable $y_r(t)$ and for any $\left| \frac{dy_r(t)}{dt} \right|$, the previous expression is obtained iif: $\left| \frac{\partial u(t)}{\partial y_r(t)} - \frac{\partial u(t-T)}{\partial y_r} \right| \to 0$ To sum up, if $\frac{du(t)}{dt}$ and T are both low, we have $\left| \frac{\partial u(t)}{\partial y_r(t)} - \frac{\partial u(t-T)}{\partial y_r} \right| \to 0$. ∎

Let us now give the new compensator.

Theorem 3.1: Reconsider the PI hysteresis model in Def. 2.2 which is rewritable as in Cons. 3.1. If the compensator is defined by:
$$u(t) = \sum_{i=0}^{n} w_i \cdot \max\left\{ \begin{array}{l} u(t-T) - r_i, \\ \min\left\{u(t-T) + r_i, y_{ei}(t-2T)\right\} \end{array} \right\} -y_r(t)$$
then $\frac{\partial y}{\partial y_r} \simeq 1$ and therefore, the hysteresis is compensated.

Proof: Replacing $u(t)$ of the model in Consequence. 3.1 by the proposed compensator in Theo. 3.1, we obtain: $y(t) = y_r(t) + O$ where $O = u(t) - u(t-T)$

$$+ \sum_{i=1}^{n} w_i \cdot \max\left\{u(t) - r_i, \min\left\{u(t) + r_i, y_{ei}(t-T)\right\}\right\}$$

$$- \sum_{i=1}^{n} w_i \cdot \max\left\{ u(t-T) - r_i, \min\left\{ \begin{array}{l} u(t-T) + r_i, \\ y_{ei}(t-2T) \end{array} \right\} \right\}$$

Knowing that the model is independant from the reference, i.e.: $\frac{\partial \sum_{i=1}^{n} w_i \cdot \max\{u(t)-r_i, \min\{u(t)+r_i, y_{ei}(t-T)\}\}}{\partial y_r} = 0$ we derive: $\frac{\partial O}{\partial y_r} = \frac{\partial u(t)}{\partial y_r(t)} - \frac{\partial u(t-T)}{\partial y_r(t)}$ which - according to Consequence. 3.2 - means $\frac{\partial O}{\partial y_r} \to 0$ Finally, we deduce that: $\frac{\partial y(t)}{\partial y_r} = \frac{\partial y_r}{\partial y_r} + \frac{\partial O}{\partial y_r} \simeq 1$ ∎

We have demonstrated that using the compensator given in Theo. 3.1, the hysteresis modelled by a classical PI technique was compensated. It is reminded that the proposed compensator contains the initial model itself (up to a signal $-u(t)$ and up to period T) according to Cons. 3.1. This means that there is no extra-calculation of the compensator since it uses the same parameters and structures than the initial model.

D. Parameters and implementation of the proposed compensator

The proposed compensator is identified and implemented as follows.

First, the hysteresis model of the process is given. It is defined by Def. 2.2. Then, the parameters r_i and w_i of the model are identified following the procedure in Section. II-C. As soon as this model is identified, the compensator is directly derived from Theo. 3.1 since they have the same parameters. This proposed compensator is implemented in cascade with the process either by direct programming or by using block diagram tool as presented in Fig. 5-a.

Remark 3.2: The implementation scheme in Fig. 5-a is equivalent to the Fig. 5-b. The difference is on the two substraction and addition blocks. The scheme in Fig. 5-b is more natural since the reference $y_r(t)$ brings in the positive input of the block.

Remark 3.3: The proposed compensator in Theo. 3.1 and presented in Fig. 5 has a (nonlinear) feedback. The structure has an inverse multiplicative form.

Remark 3.4: As we can see, an additional one period delay appears in the proposed compensator in Theo. 3.1 (see for example $u(t)$ and $u(t-T)$). It ensures that no algebraic loop is in the feedback of the compensator, and therefore no error occurs during functioning. In the block diagram implementation, this delay is obtained using the delay-block (Fig. 5).

IV. EXPERIMENTAL RESULTS

In this section, we apply the proposed compensator to feedforward control the bending of a piezoactuator.

A. The experimental setup

The piezoactuator used in the experiments is a unimorph cantilever with rectangular cross-section. It is made up of one piezoceramic layer (PZT-151) and one passive layer (Nickel). When applying a voltage to the piezolayer, it expands/contracts resulting a bending of the whole cantilever (Fig. 6-a). The setup - pictured in Fig. 6-b - is composed of:
- the unimorph piezoelectric cantilever with dimensions: $15mm \times 2mm \times 0.3mm$, where $0.2mm$ and $0.1mm$ are the thicknesses of the PZT and of the Nickel respectively,
- a computer and a dSPACE-board that is used to acquire the measurements and to provide the control signal $u(t)$ and reference $y_r(t)$. The software MATLAB-SIMULINK is used for that. The refresh frequency of the acquisition material is $f_s = 5kHz$ ($T = 0.2ms$) which is high enough relative to the frequencies of the signal to be used,
- a high voltage (HV) amplifier,
- and an optical sensor from *Keyence* (LC2420) with a resolution up to $10nm$. This sensor is used to report the bending $y(t)$ of the actuator for the identification and for the validation aspects.
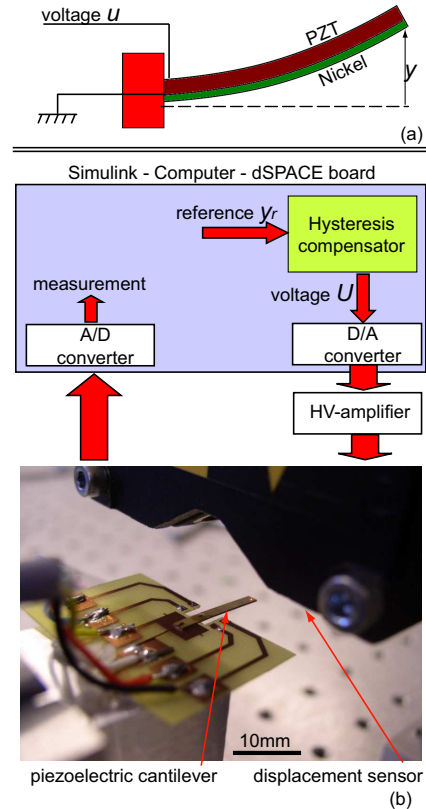

Fig. 6. Photography of the piezoelectric actuator.

B. Modeling and parameters identification

The piezoactuator has a strong hysteresis. It is modeled using the classical PI approach described by Def. 2.2 and by Fig. 2. To identify the parameters r_i and w_i, we follow the procedure in Section. II-C.

First, a sine input voltage $u(t)$ is applied to the piezoactuator. The amplitude of the sine signal - equal to $u_A = 80V$ - corresponds to a bending that covers the required range (nearly $20\mu m$ for us) in the application. The working frequency is chosen to correspond to that required by the application: $f = 0.1Hz$.

After reporting the measured bending $y(t)$, we plot $y(t)$ versus $u(t)$ and shift it in order to obtain a shifted map (u, y) in the positive section like in Fig. 3. Then, we define the number of backlashes n. The choice is a compromise because a low number generates a less accurate model while a high number increases its complexity. In our case, we choose $n = 15$.

Then, we split the range $160V$ $(= 2 \cdot u_A)$ into 16 $(= n+1)$ partitions and we compute the bandwidths bw_i. Using the bandwidths and the ascending curve of the shifted $y(t)$, the discrete values y_i are deduced. The matrix A is also computed from the derived bw_i and from (equ 1) and (equ 2).

Finally, we derive the thresholds using $r_i = \frac{bw_i}{2}$ and the weighting w_i using (equ 3).
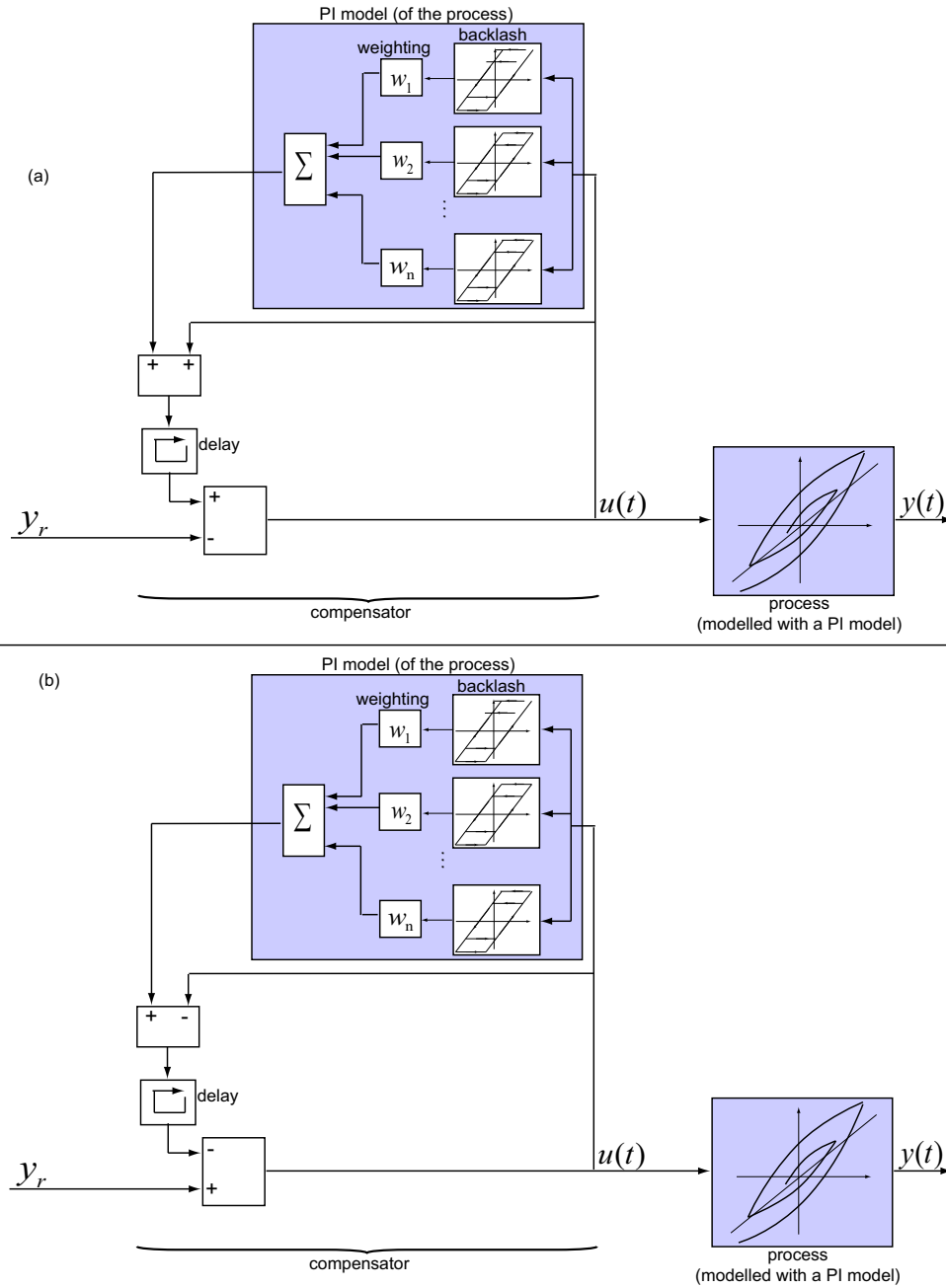
Fig. 5. Diagram showing the implementation of the proposed compensator. The two schemes (a) and (b) are equivalent.

Using different amplitude of sine input $u(t)$, the simulation of the identified model is now compared with the experimental result. Fig. 7 shows that the identified model captures the hysteresis behavior of the piezoactuator. This figure also shows that the hysteresis amplitude is about 23% $(= \frac{h}{H} \approx \frac{10\mu m}{43\mu m})$.

C. Results with the compensator

We now implement the compensator pictured in Fig. 5-b. When applying a sine input reference $y_r(t)$ with the working frequency $f = 0.1Hz$, we see that the map

(y_r, y) is linear and with a unity slope (Fig. 8-a) and thus the accuracy of the controlled system is obtained. Fig. 8-b plots the corresponding tracking response. Finally, the tracking error is plotted in Fig. 8-c. As pictured in the figure, the maximal tracking error $(y_r - y)$ is nearly $0.5\mu m$ which is negligible relative to the operational range. These results demonstrate that the proposed compensator reduces the hysteresis from 23% to less than 2.5% $(\approx \frac{1\mu m}{40\mu m}, 1\mu m$ and $40\mu m$ being the range of the error and the operational range respectively).
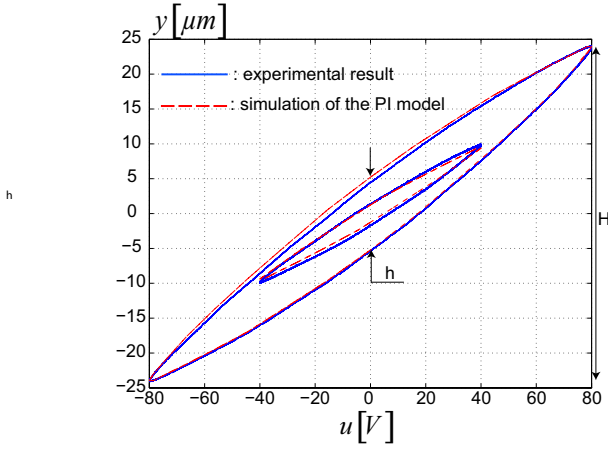
Fig. 7. The hysteresis of the piezoactuator: experimental result and simulation of the identified PI model.
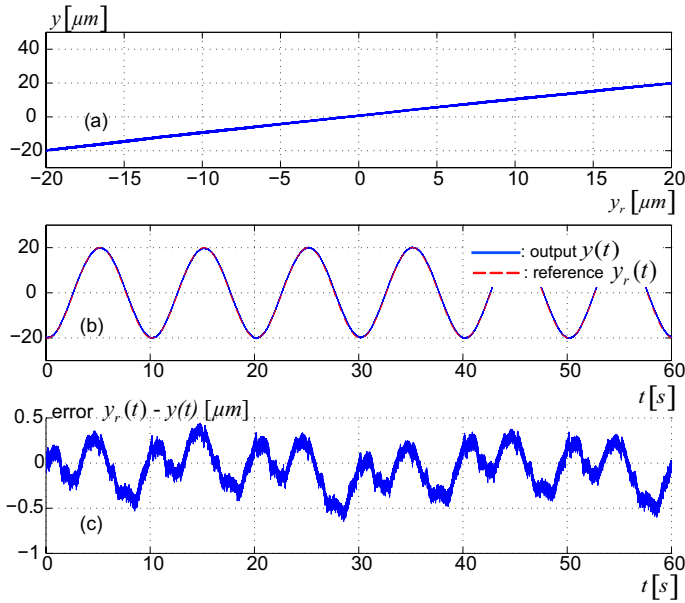

Fig. 8. Experimental result when using the proposed compensator.

V. CONCLUSION

A new compensator technique for the classical Prandtl-Ishlinskii (PI) hysteresis model was proposed in this paper. The main particularity of the proposed technique is that no additional computation is required for the compensator. As soon as the model is identified, the compensator is obtained. The approach is dedicated to compensate static hysteresis in smart materials such as piezoceramics. The experimantal results on piezoactuators demonstrated the efficiency of the proposed method. Future works include the extension of the proposed compensation technique to multivariable hysteresis compensation. Application of the proposed method in piezoactuators working in tasks where the reference input is more complicated (varying amplitude, etc) will also be considered.

REFERENCES

[1] A. Bergander, W. Driesen, T. Varidel, M. Meizoso and J. M. Breguet, "Mobile $cm3$-microrobots with tools for nanoscale imaging and micromanipulation" Mechatronics & Robotics, pp.1041-1047, 13-15 Aachen, Germany, September 2004.
[2] M. Rakotondrabe, Y. Haddab and P. Lutz, "Development, modelling and control of a micro/nano positioning 2DoF stick-slip device", IEEE/ASME Trans. on Mechatronics, pp:733-745, Dec 2009.
[3] G. Bining, C. F. Quate and Ch. Berger, "Atomic Force Microscope", Physical Review Letters, 56, pp.930-933, 1986.
[4] J. Agnus, J. M. Breguet, N. Chaillet, O. Cois, P. de Lit, A. Ferreira, P. Melchior, C. Pellet and J. Sabatier, "A smart microrobot on chip: design, identification and modeling", IEEE/ASME Int. Conf. on Advanced Intelligent Mechatronics, pp.685-690, July 2003.
[5] M. Rakotondrabe and Ioan A. Ivan, "Development and Force/Position Control of a New Hybrid Thermo-Piezoelectric microGripper dedicated to automated pick-and-place tasks", IEEE Trans ASE, Oct 2011.
[6] K. K. Leang and S. Devasia, "Hysteresis, creep, and vibration compensation", IFAC Conference on Mechatronic Systems (Mech), pp.283-289, Berkeley CA USA, December 2002.
[7] M. Rakotondrabe, Y. Haddab and P. Lutz, "Quadrilateral modeling and robust control of a nonlinear piezoelectric cantilever", IEEE Trans CST, 17(3), pp.528-539, May 2009.
[8] J. Agnus and N. Chaillet, "Dispositif de commande d'un actionneur pizolectrique et scanner muni de ceux-ci", INPI Patent, n^o FR03000532, 2003.
[9] Fleming, A. J. and Moheimani, S. O. R., "A grounded load charge amplifier for reducing hysteresis in piezoelectric tube scanners," Review of Scientific Instruments, 76(7), 073707(1-5), July 2005.
[10] G. M. Clayton, S. Tien, A. J. Fleming, S. O. R. Moheimani, S. Devasia, "Inverse-feedforward of charge-controlled piezopositioners", Mechatronics, V.(5-6), page 273-281, June 2008.
[11] M. Rakotondrabe, "Bouc-Wen modeling and inverse multiplicative structure to compensate hysteresis nonlinearity in piezoelectric actuators", IEEE Trans ASE, April 2011.
[12] Saeid Bashash and Nader Jalili, "A Polynomial-Based Linear Mapping Strategy for Feedforward Compensation of Hysteresis in Piezoelectric Actuators", ASME J. of Dynamic Syst. Meas. and Control, 130(3), May 2008.
[13] K. Kyle Eddy, "Actuator bias prediction using lookup-table hysteresis modeling", US Patent-08/846545, February 1999.
[14] D. Croft, G. Shed and S. Devasia, "Creep, hysteresis and vibration compensation for piezoactuators: atomic force microscopy application", ASME Journal of Dynamic Systems, Measurement and Control, 123(1), pp.35-43, Mars 2001.
[15] A. Dubra and J. Massa and C.l Paterson, "Preisach classical and nonlinear modeling of hysteresis in piezoceramic deformable mirrors", Optics Express, 13(22), pp.9062-9070, 2005.
[16] M. Rakotondrabe, C. Clévy and P. Lutz, "Complete open loop control of hysteretic, creeped and oscillating piezoelectric cantilevers", IEEE Trans. ASE, 7(3), pp:440-450, July 2009.
[17] W. T. Ang, P. K. Kholsa and C. N. Riviere, "Feedforward controller with inverse rate-dependent model for piezoelectric actuators in trajectory-tracking applications", IEEE/ASME Transactions on Mechatronics, 12(2), pp.134-142, April 2007.
[18] B. Mokaberi and A. A. G. Requicha, "Compensation of scanner creep and hysteresis for AFM nanomanipulation", IEEE Trans ASE, 5(2), pp.197-208, April 2008.
[19] M. A. Krasnosel'skii and A. V. Pokrovskii, "Systems with hysteresis", Springer-Verlag, Berlin, 1989.
[20] K. Kuhnen and H. Janocha, "Inverse feedforward controller for complex hysteretic nonlinearities in smart-materials systems", Control of Intelligent System, Vol.29(3), pp.74-83, 2001.