

Inversion-Free Feedforward Dynamic Compensation of Hysteresis Nonlinearities in Piezoelectric Micro/Nano-Positioning Actuators

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Abstract—A new methodology that employs the rate-dependent Prandtl-Ishlinskii model (RDPI) as a model and a compensator is suggested in this study for modeling and compensation of rate-dependent hysteresis nonlinearities of a piezoelectric actuator. The technique employs a restructuration of the model that ignores the need to derive an inverse model, which avoids the additional calculations required to formulate a compensator. The simulation results are presented to demonstrate the effectiveness of the strategy on modeling and compensation of hysteresis nonlinearities at various frequencies. The simulation results were followed by experimental study on a piezoelectric actuator that exhibits rate-dependent hysteresis nonlinearities. The results demonstrate that the proposed methodology can be employed effectively for compensation of rate-dependent hysteresis nonlinearities without developing an inverse model.

I. INTRODUCTION

Smart material-based actuators are increasingly being explored for micro- and nano-positioning applications as well as for manipulation in sub-nano meters [1]. These actuators are also popular for designing precise positioners due to their high resolution [2]-[3]. However, the advantages of these actuators are limited to a narrow range of operating conditions due to the presence of hysteresis nonlinearities, which cause response oscillations in the open-loop systems, lack of tracking performance and instabilities in the closed-loop systems [4]. Designing controllers able to improve the tracking performance of these actuators requires an accurate model that can describe the hysteresis properties accurately. Consequently, different models have been proposed in the literature to predict hysteretic behaviour of smart actuators, these include the Preisach model [4], the Prandtl-Ishlinskii model [5-9], the Bouc-Wen model [10], and the Duhem model [11]. Among the available models, Prandtl-Ishlinskii model is considered attractive for real-time applications due to its accuracy and ease of implementation. This model has been widely employed for modeling and compensation of hysteresis nonlinearities of smart actuators, see for example [5-9].

Different controller designs have been suggested in the literature in order to compensate for the hysteresis effects

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of smart material-based actuators. These techniques fall in two categories inverse-based hysteresis compensation methods and model-based hysteresis compensation methods. In inverse-based hysteresis compensation a cascade arrangement of a hysteresis model and its inverse are employed together. Application of these methods requires formulating an inverse for the hysteresis model itself, which is a challenging task [4]. The model-based methods employ the hysteresis model to design nonlinear feedback controllers able to compensate for the actuator hysteresis. These methods include hybrid control [12], adaptive control [13], and energy-based [14] control, that develop a controller on the basis of the hysteresis model itself. Most of these methods, however, lack the consideration of the frequency effects which become significant at high excitations of input rate.

Recently, a rate-dependent version of the Prandtl-Ishlinskii (RDPI) model has been suggested with the corresponding rate-dependent inverse model [6]. The model and its inverse were efficiently applied to piezoelectric actuators [9]. The RDPI model is a hysteresis model that tracks the experimental hysteresis at low and at high frequencies. The advantage of this model is that it can account for the hysteresis nonlinearities and the dynamics of the actuator and the plant in one single nonlinear model. The RDPI compensator [6] is the inverse of the RDPI model, where the parameters of the operators are dependent on the excitation frequency of the driving input (for the model) and dependent on the frequencies of the reference input (for the compensator). However, not all RDPI models can be inverted to derive a RDPI compensator [6]. Some conditions on the parameters have to be satisfied which makes this technique slightly restricted [15, 16].

In this paper, a new RDPI compensator is proposed. The compensator is not obtained from the inversion of the RDPI model, hence the previous restriction is bypassed. Based on a restructuration of the model with the inverse multiplicative scheme, the compensator is directly yielded as soon as this model is identified. The inversion is therefore avoided and no additional calculation is required.

The contributions of the paper can be summarized as

- a new compensation technique to reduce the rate-dependent hysteresis in smart material-based actuators at different excitation frequencies without using model inversion,
- and simulation and experimental case: utilization of the proposed scheme for a piezoelectric actuator. Experimental results are provided to demonstrate the effectiveness of the suggested approach.

The paper is presented as follows. In Section II, the RDPI model is revisited. Section III presents the inverse multiplicative scheme as a technique for compensation of rate-dependent hysteresis nonlinearities with a numerical example. An experimental study on compensation of rate-dependent hysteresis nonlinearities of piezoelectric actuator using the inverse multiplicative method is presented in Section IV. The conclusions of the paper are summarized in Section V.

II. BACKGROUND

A. General scheme

Fig. 1 illustrates a control system consists of an actuator typified by hysteresis nonlinearities that can be represented by Prandtl-Ishlinskii model, and a compensator for cancelling the hysteresis nonlinearities out. u in this scheme represents the command (driving) input signal, y_r is the input of the compensator (the reference input), and y is the output of the inverse compensation when system controlled in a feed-forward manner. The smart actuators are generally integrated in a plant that exhibits low dynamics such that the relation between the output y and the input u can be represented by rate-dependent hysteresis which is strongly dependent on the excitation frequency of the applied input. This study aims at modeling and compensation of hysteresis of a plant that exhibits dynamic hysteresis nonlinearities between the input u and the output y .

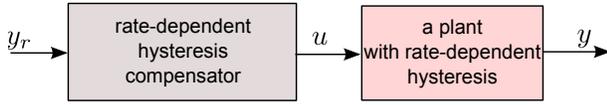


Fig. 1. A plant with rate-dependent hysteresis and a feedforward hysteresis compensator.

B. The RDPI model

A rate-dependent version of the Prandtl-Ishlinskii (RDPI) model has been developed recently for modeling dynamic hysteresis nonlinearities of smart material-based actuators [6]. The suggested model was applied in other studies to describe the (voltage-displacement) hysteresis loops of piezoelectric actuators and (current-displacement) loops of magnetostrictive actuators. The mathematical formulation of the model is revisited in this section. This model is employed in this brief to describe the rate-dependent hysteresis nonlinearities as well as to design rate-dependent compensator without formulating an inverse model.

Throughout the text, we deal with real absolutely continuous functions defined in the interval $(0, T)$. The space of such functions is denoted by $AC(0, T)$. For the input signal $u(t) \in AC(0, T)$ and for $i = 1, 2, \dots, n$, where n is an integer, let $r_i(\dot{u}(t)) \in AC(0, T)$ be given functions such that

$$r_n(\dot{u}(t)) \geq r_{n-1}(\dot{u}(t)) \geq \dots \geq r_1(\dot{u}(t)) > 0.$$

The output of the play operator is $\xi_i(t) = \Phi_{r_i(\dot{u}(t))}[u](t)$. For inputs and thresholds that are piecewise linear, that is, linear in each interval of a partition $0 = t_0 < t_1 < \dots < t_l = T$, the output of the play operator can be expressed for $t \in [t_{j-1}, t_j)$

$$\xi_i(t) = \max\{u(t) - r_i(\dot{u}(t)), \min\{u(t) + r_i(\dot{u}(t)), \xi_i(t_{j-1})\}\}, \quad (1)$$

with initial condition

$$\xi_i(0) = \max\{u(0) - r_i(\dot{u}(0)), \min\{u(0) + r_i(\dot{u}(0)), 0\}\}.$$

Thus, the output of the RDPI hysteresis model $y(t) = \Gamma[u](t)$ is expressed as

$$y(t) = \Gamma[u](t) := a_0 u(t) + \sum_{i=1}^n a_i \Phi_{r_i(\dot{u}(t))}[u](t), \quad (2)$$

where a_0 and a_i are positive constants.

C. The discrete form

The discrete form of the RDPI model can be expressed for an input $u(t)$ with sampling time $T_s = t_k - t_{k-1}$, where $k = 1, 2, 3, \dots$, as

$$y(k) = \Gamma[u](k) := a_0 u(k) + \sum_{i=1}^n a_i \Phi_{r_i(v(k))}[u](k), \quad (3)$$

where

$$\xi_i(k) = \max\{u(k) - r(v(k)), \min\{u(k) + r(v(k)), \xi_i(k-1)\}\} \quad (4)$$

and

$$v(k) = \frac{u(k) - u(k-1)}{T_s}. \quad (5)$$

III. AN INVERSION-FREE FEEDFORWARD RATE-DEPENDENT COMPENSATOR

A. The compensator

The proposed rate-dependent feedforward compensator is presented in this section. The compensator is built based on the inverse multiplicative scheme that employs a restructuring of the RDPI model itself. Consequently, no additional calculation is required to derive the inverse of the proposed model, which implies that the compensator is yielded as soon as the model is identified. In addition, unlike the available strategies, no condition has to be satisfied in order to ensure the invertibility of the model, which offers more flexibility in formulating the rate-dependent model.

For an input function $u(k)$ monotone (non-decreasing or non-increasing) in each interval $[t_k, t_{k-1}]$ of a partition $0 = t_1 < \dots < t_K = T$, and $a_0 > 0$, the output of the RDPI model for $k > 1$ with

$$u(k) = \Psi[y_r(k)] = a_0^{-1} \left(y_r(k) - \Omega[u](k-1) \right), \quad (6)$$

where

$$\Omega[u](k-1) = \sum_{i=1}^n a_i \Phi_{r_i(v(k-1))}[u](k-1), \quad (7)$$

is $y(k) \cong y_r(k)$, and the compensation error that is defined as $e(k) = y_r(k) - y(k)$ yields $e(k) \rightarrow 0$. Consequently,

Equation (6) is a RDPI compensator for the discrete RDPI model defined in Equation (3).

As we can observe, the hysteresis presented by the RDPI model in Equation (3) can be compensated using the model itself and a restructuring of this latter. The main advantage that this approach is the availability of the rate-dependent compensator as long as the RDPI is formulated. The compensator itself can be directly obtained without additional calculation to formulate an inverse rate-dependent model. Thus, the parameters of the rate-dependent compensator are similar to those of the rate-dependent model.

The block diagram of the suggested methodology is shown in Fig. 2. As the figure illustrates, the suggested compensator can be represented by an inverse multiplicative scheme that compensates for the rate-dependent hysteresis nonlinearities in a feedforward open-loop manner. The sampling time T_s block in the figure stands for the delay between the control input signal $u(t)$ and the one used by the compensator.

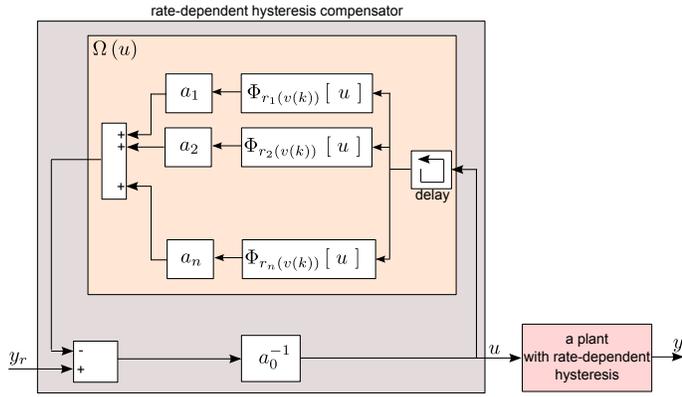


Fig. 2. The block diagram of open-loop compensation using inverse multiplicative structure.

B. Illustrative example

A reference input signal $y_r(k) = 10 \sin(2\pi f k T_s)$ where $f = 1, 25,$ and 50 Hz, was applied to a RDPI model formulated using $n = 5$ play operators, $a_0 = 0.729$, $a_1 = 0.211$, $a_2 = 0.113$, $a_3 = 0.061$, $a_4 = 0.032$, $a_5 = 0.071$, $\delta_1 = 1.750$, and $\delta_2 = 3.762 \times 10^{-4}$. The suggested compensator in Equation (6) was formulated with sampling time $T_s = 10^{-3}$ s, 10^{-4} s, and 10^{-6} s to reduce the rate-dependent nonlinearities of the RDPI model.

The output-input hysteresis nonlinearities of the RDPI model are illustrated in Figure 3(a), (b), and (c) at excitation frequency of 1, 25, and 50 Hz and sampling time of $T_s = 10^{-3}$ s. The time history of the compensation error $e(k)$ is illustrated in Figure 3(d), (e), and (f) at different excitations of frequency. Each excitation was applied at three different sampling of time T_s . The simulation results demonstrate that reducing the sampling time improve the compensation results. The output-input mapping (y_r, y) is displayed in Figure 4 at excitation frequency of 50 Hz applied at two different sampling of time $T_s = 10^{-3}$ s and $T_s = 10^{-4}$ s.

Finally, Figure 5 shows the output-input mapping (y_r, y) at $T_s = 10^{-6}$ s and three different excitations of frequency. The simulation results illustrate that complete compensation for all excitations can be obtained when the sampling time ($T_s = 10^{-6}$ s).

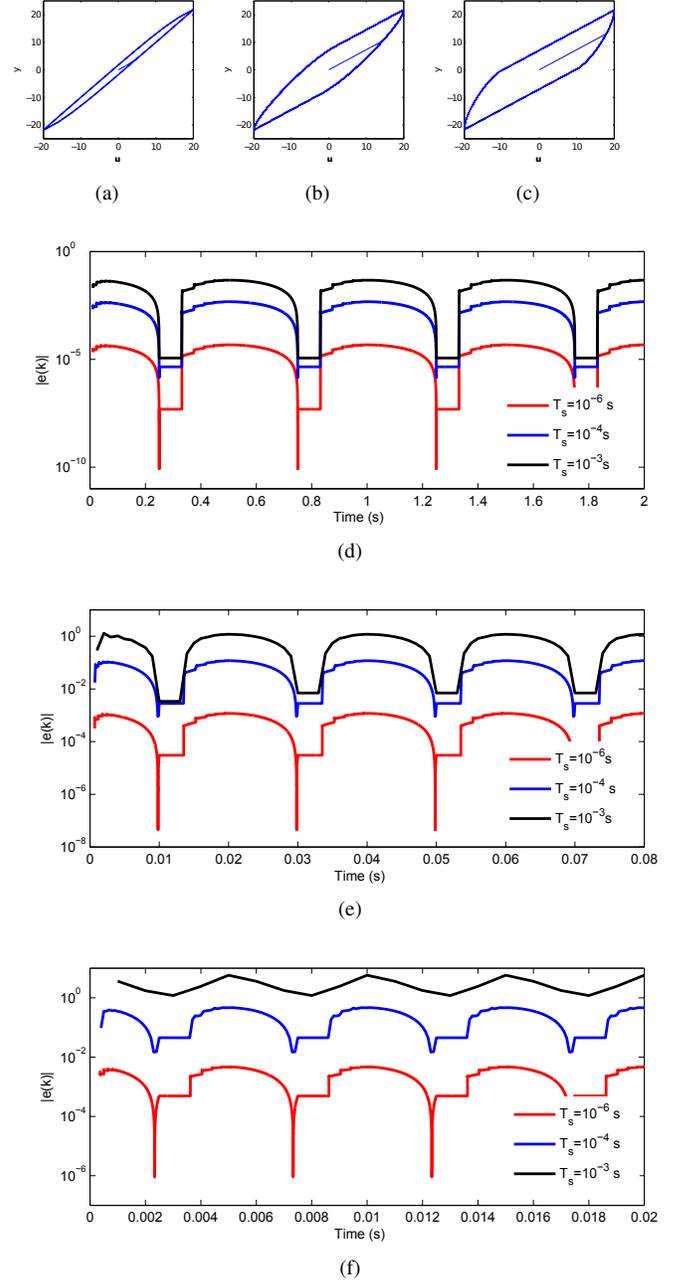


Fig. 3. The output of the RDPI model without compensation at excitation frequency of (a) 1 Hz, (b) 25 Hz, and (c) 50 Hz. The time history of the error between the reference input y_r and the RDPI model output y when the hysteresis is reduced using the compensator of Equation (6) with (d) $T_s = 10^{-3}$ s and the reference input y_r of 1 Hz, (e) with $T_s = 10^{-4}$ s and the reference input y_r of 25 Hz, and (f) with $T_s = 10^{-6}$ s and the reference input y_r of 50 Hz.

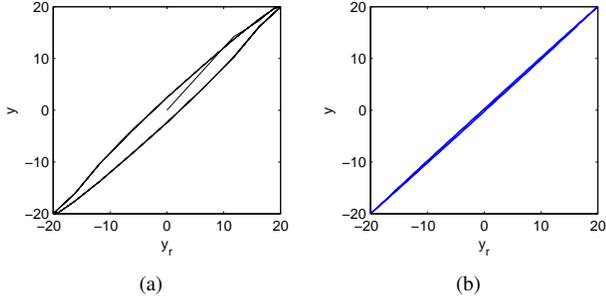


Fig. 4. The reference input y_r versus the output y of the RDPI model when the compensator in Equation (6) is applied at excitation frequency of 50 Hz with (a) $T_s = 10^{-3}$ s, and (b) $T_s = 10^{-4}$ s.

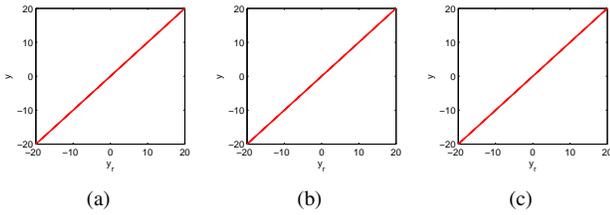


Fig. 5. The reference input y_r versus the output y of the RDPI model when the compensator in Equation (6) is applied at $T_s = 10^{-6}$ s with the reference input of (a) 1 Hz, (b) 25 Hz, and (c) 50 Hz.

IV. EXPERIMENTAL APPLICATION TO A PIEZOELECTRIC ACTUATOR

A. Presentation of the experimental platform

In order to validate the effectiveness of the proposed algorithm in compensation of rate-dependent hysteresis nonlinearities, an experimental study was carried out in the laboratory with a piezoelectric actuator. These actuators have been considered for applications require high resolution, high bandwidth and high stiffness. The proposed actuator is a cantilevered type, employed widely for micromanipulation and assembly of objects in micro/nano scale levels [1]. This kind of actuators are also employed for scanning in atomic force microscopy (AFM) [17], actuation in miniaturized bio-inspired robots [18] as well as for micromirror orientation [19, 20]. However, similar to other kinds of smart material based actuators, piezoelectric actuators exhibit strong rate-dependent hysteresis nonlinearities that are significant at high excitation of input frequency [21, 22].

A piezoelectric actuator with several layers (multilayer) was used in the experimental study. The actuator has a high range of bending y at low voltages u . The layers that the actuator integrates are based on lead zirconate titanate (PZT) ceramics and are glued together. Imposing an input voltage $u(t)$ to the layers of the actuator results a bending (displacement) $y(t)$ (Fig. 6-a). A schematic representation of experimental platform setup is presented in Fig. 6-b and is consisted of:

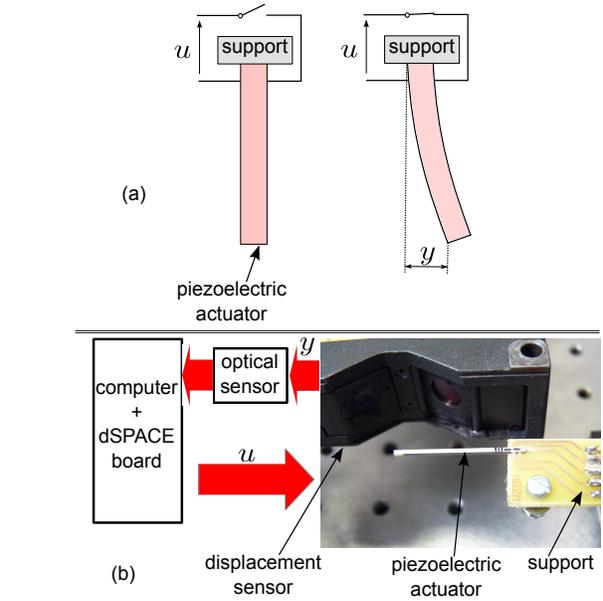


Fig. 6. The experimental platform: (a) the piezoelectric actuator, and (b) schematic representation of the setup.

- a multilayered piezoelectric cantilevered actuator (with 36 layers) having total and active dimensions (active length \times width \times thickness) of: $15\text{mm} \times 2\text{mm} \times 2\text{mm}$,
- an optical displacement sensor used to measure the bending of the actuator. The sensor (LC2420 from KEYENCE) is tuned to have a resolution of 10nm , and a precision of 100nm with a bandwidth more than 1kHz ,
- a computer with MATLAB-SIMULINK software used to handle the different signals (input voltage, reference input, output displacement) and to formulate the proposed compensator,
- and a dSPACE board that serves as DAC/ADC converter between the computer and the rest of the experimental setup. The refresh time of the computer and dSPACE board is set to $50\mu\text{s}$ which permits to account for the bandwidth of the actuator.

B. Characterization

Measured hysteresis loops obtained from a piezoelectric actuator are shown in Figure 7. These measurements are used to identify the parameters of the RDPI model. The rate effect of the applied input can be integrated in the dynamic threshold function as

$$r_i(v(k)) = \delta_1 i + \delta_2 |v(k)|, \quad (8)$$

where δ_1 and δ_2 are positive constants. In order to identify the RDPI model expressed in Equation (3) with the threshold function in Equation (8), the hysteresis characterization error can be defined as

$$e_c(k) = \Psi[u](k) - y(k), \quad (9)$$

where $y(k)$ is the measured output displacement of the piezoelectric cantilevered actuator when an excitation input voltage $u(k)$ is applied at a certain excitation frequency, and

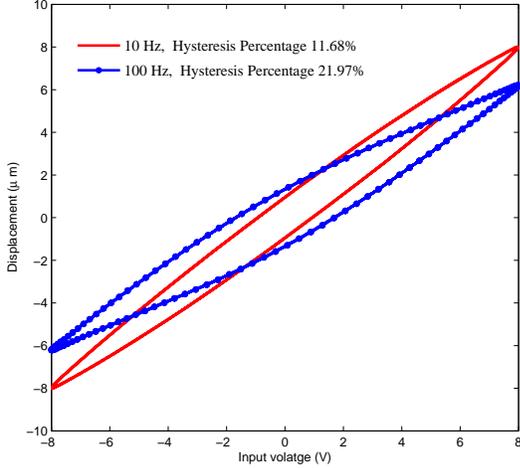


Fig. 7. The measured hysteresis loops of the piezoelectric actuator at excitation frequency of 10 Hz and 100 Hz.

$\Psi[u](k)$ is the output of the RDPI model under same input voltage. The index k ($k = 1, 2, \dots, K$) refers to the number of data points $K = 400$ considered in computing the error for one complete hysteresis loop. The parameter vector $\Pi = \{ \delta_1, \delta_2, a_0, a_1, a_2, \dots, a_{10} \}$ of the RDPI model Ψ , was subsequently identified for different excitations of input frequency through minimization of least-square errors as

$$\min \left(\sum_{k=0}^K e_c^2(k) \right) = \min \left(\sum_{k=0}^K (\Psi[u](k) - y(k))^2 \right), \quad (10)$$

where $\Psi[u](k)$ is the discrete form RDPI model expressed in Equation (3).

The optimization problem was solved using MATLAB constrained optimization toolbox such that $\delta_1 > 0$ and $\delta_2 > 0$. The number of the RDP operators n has been chosen as $n = 10$ which is sufficient to formulate a RDPI model can characterize the measured displacement presented in Figure 7. The identified parameters were found as: $\delta_1 = 1.0136$, $\delta_2 = 3.948 \times 10^{-4}$, $a_0 = 0.621$, $a_1 = 0.135$, $a_2 = 0.1079$, $a_3 = 0.0862$, $a_4 = 0.0689$, $a_5 = 0.0550$, $a_6 = 0.0439$, $a_7 = 0.0351$, $a_8 = 0.0280$, and $a_9 = 0.0224$, and $a_{10} = 0.0179$. Following to the proposed method in section-III-A, the identified model is directly employed to yield the corresponding RDPI compensator. The model and compensator are applied in the next section for compensation of rate-dependent hysteresis of the piezoelectric actuator.

V. COMPENSATION RESULTS

In this section, the compensator proposed in section-III-A is implemented for compensation of hysteresis nonlinearities of the piezoelectric actuator at different excitations of input frequency. Based on the suggested algorithm no extra-calculations are required to calculate the compensator which can be directly obtained using the restructuring of the initial RDPI model that has been identified in the

previous section. Furthermore, unlike the classic approach, no specific condition has to be satisfied in order to ensure the invertibility of the RDPI model.

In order to compare the efficiency of the proposed compensator with the classical compensation technique, the compensation results using the classic technique was obtained. The classic RDPI compensator represents the exact inverse $\Psi^{-1}[y_r](t)$ of the RDPI model $\Psi[u](t)$ [6] such that $y(k) = y_r(k)$.

Figure 8(a) shows the results of compensation obtained using the proposed method whilst those in figure 8(b) are compensation results when the classical method in [6] is employed. The figures display the time history of the output displacement y for both compensators under a desired displacement y_r of $40\mu m$ amplitude applied at excitation frequency of 80 Hz. Figure 8(c) shows the error of compensation $y_r - y$ which is bounded within $3\mu m$ over a desired output displacement of $40\mu m$. These results demonstrate that the proposed compensation technique permits a nice tracking performance for a wide range of input frequency, which is similar to the results obtained using the existing method. However, unlike the classic approach, the new technique avoids the extra-calculations required to obtain the inverse model. Furthermore, no specific condition has to be satisfied to ensure the invertibility of the RDPI model as in the existing method, which permits more flexibility in formulating the RDPI model.

VI. CONCLUSIONS

A new feedforward compensator constructed based on the RDPI (rate-dependent Prandtl-Ishlinskii) model is presented to compensate for rate-dependent hysteresis nonlinearities without formulating an inverse rate-dependent model. The methodology employs the RDPI model to characterize the hysteresis nonlinearities as well as to compensate for the rate-dependent hysteresis. The proposed RDPI compensator is constructed with a linear reversible term and a rate-dependent hysteretic term in an inverse multiplicative scheme. The main advantage is that as soon as the RDPI model is available, the compensator is yielded by structure without extra-calculation to obtain the compensator parameters. Furthermore, no condition must be satisfied to ensure the invertibility of the model, contrary to the classical method. The experimental results show that the suggested feedforward controller can effectively compensate for the rate-dependent hysteresis of piezoelectric micropositioning actuator at different excitation frequencies. Future work will include conducting the experimental tests with the proposed compensator over a wide range of operating conditions such as various levels of input amplitude and different excitations of input frequency. In addition, the parameters uncertainty corresponds to the RDPI model with the proposed compensator will be investigated.

ACKNOWLEDGMENT

This work is supported by the national ANR-JCJC C-MUMS-project (National young investigator project ANR-

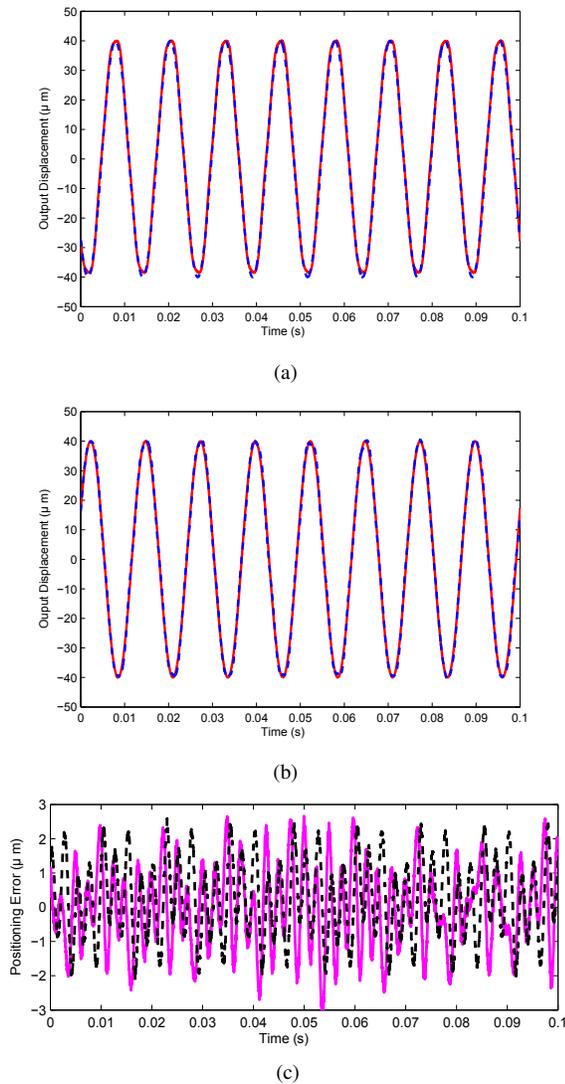


Fig. 8. (a) time history of the desired input displacement y_r and the output displacement y using the proposed compensator at excitation frequency of 80 Hz, (b) the time history of the desired input displacement y_r and the output displacement y when using the classical methodology [6] at excitation frequency of 80 Hz, and (c) the time history of the compensation error when both the proposed and the classical method applied at excitation frequency of 80 Hz.

12-JS03007.01: Control of Multivariable Piezoelectric Microsystems with Minimization of Sensors).

REFERENCES

- [1] J. Agnus, N. Chaillet, C. Clévy, S. Dembélé, M. Gauthier, Y. Haddab, G. Laurent, P. Lutz, N. Piat, K. Rabenorosoa, M. Rakotondrabe, and B. Tamadazte, "Robotic microassembly and micromanipulation at FEMTO-ST," *Journal of Micro-Bio Robotics*, vol. 8, no. 2, pp. 91-106, 2013.
- [2] M. Rakotondrabe, Y. Haddab, and P. Lutz, "Plurilinear modeling and discrete -synthesis control of a hysteretic and creeped unimorph piezoelectric cantilever," *Proceedings of International Conference on Automation, Robotics, Control and Vision*, pp. 1-8, Singapore, 2006.
- [3] X. Wang, G. Alici, and X. Tan, "Modeling and inverse feedforward control for conducting polymer actuator with hysteresis," *Smart Materials and Structures*, vol. 23, no. 4, pp. 25015-25023, 2014.

- [4] M. Rakotondrabe, *Smart materials-based actuators at the micro/nano-scale: characterization, control and applications*, Springer-Verlag, New York, 2013.
- [5] M. Al Janaideh, "About the output of the inverse compensation of the Prandtl-Ishlinskii model," *Proceedings of American Control Conference*, pp. 247-252, Washington, DC, USA, 2013.
- [6] M. Al Janaideh and P. Krejčí, "Inverse rate-dependent Prandtl-Ishlinskii model for feedforward compensation of hysteresis in a piezomicropositioning actuator," *IEEE/ASME Transactions on Mechatronics*, vol. 18, no. 5, pp. 1498-1507, 2013.
- [7] M. Rakotondrabe, C. Clévy, and P. Lutz, "Complete open loop control of hysteretic, creeped, and oscillating piezoelectric cantilevers," *IEEE Transactions on Automation Science and Engineering*, vol.7, no. 3, pp. 440-450, 2010.
- [8] M. Rakotondrabe, "Classical Prandtl-Ishlinskii modeling and inverse multiplicative structure to compensate hysteresis in piezoactuators", *Proceedings of American Control Conference*, pp. 1646-1651, Montreal, Canada, 2012.
- [9] O. Aljanaideh, M. Al Janaideh, and M. Rakotondrabe, "Enhancement of micro-positioning accuracy of a piezoelectric positioner by suppressing the rate-dependant hysteresis nonlinearities", *Proceedings of IEEE/ASME International Conference on Advanced Intelligent Mechatronics*, pp. 1683-1688, Besancon, France, 2014.
- [10] M. Rakotondrabe, "Bouc-Wen modeling and inverse multiplicative structure to compensate hysteresis nonlinearity in piezoelectric actuators," *IEEE Transactions on Automation Science and Engineering*, vol. 8, no. 2, pp. 428-431, 2011.
- [11] J. Oh and D. S. Bernstein, "A semilinear Duhem model for rate-independent and rate-dependent hysteresis," *IEEE Transactions on Automatic Control*, vol. 50, no. 5, pp. 631-645, 2005.
- [12] M. Al Janaideh, R. Naldi, L. Marconi, and P. Krejčí, "A hybrid system for a class of hysteresis nonlinearity: modeling and compensation" *Proceedings of IEEE Conference on Decision and Control*, Maui, HI, pp. 5380-5385, 2012.
- [13] M. Al Janaideh and D. S. Bernstein, "Inversion-free adaptive control of uncertain systems with SMA actuation," *Proceedings of the American Control Conference*, pp. 3585-3590, Washington, DC, 2013.
- [14] R. Gorbet, K. Morris, and D. Wang, "Passivity-based stability and control of hysteresis in smart actuators," *IEEE Transactions on Control Systems Technology*, vol. 9, no. 1, pp. 5-16, 2001.
- [15] M. Al Janaideh and P. Krejčí, "An inversion formula for a Prandtl-Ishlinskii operator with time dependent thresholds," *Physica B*, vol. 406, no. 8, pp. 1528-1532, 2011.
- [16] P. Krejčí, M. Al Janaideh, and F. Deasy, "Inversion of hysteresis and creep operators," *Physica B*, vol. 407, no. 8, pp. 1354-1356, 2012.
- [17] G. Bining, C. F. Quate and C. Berger, "Atomic force microscope," *APS Physical Review Letters*, vol. 56, no. 9, pp. 930-933, 1986.
- [18] R. Wood, J. Whitney, and B. Finio, "Mechanics and actuation for flapping-wing robotic insects," Published Editors: R. Blockley and W. Collection, *Encyclopedia of Aerospace Engineering*, pp. 4393-4406, 2011.
- [19] Y. Zhu, W. Liu, K. Jia, W. Liao, and H. Xie, "A piezoelectric unimorph actuator based tip-tilt-piston micromirror with high fill factor and small tilt and lateral shift," *Sensors and Actuators A: Physical*, vol.167, no. 2, pp. 495-501, 2011.
- [20] S. Lescano, D. Zlatanov, M. Rakotondrabe and N. Andreff, "Kinematic analysis of a meso scale parallel robot for laser phonomicrosurgery", *Proceedings of International Conference on Interdisciplinary Applications in Kinematic*, Lima, Peru, September 2013.
- [21] M. Al Janaideh, "Generalized Prandtl-Ishlinskii hysteresis model and its analytical inverse for compensation of hysteresis in smart actuators," PhD thesis, Department of Mechanical and Industrial Engineering, Concordia University, Canada, 2009.
- [22] M. Al Janaideh and P. Krejci, "Prandtl-Ishlinskii hysteresis models for complex time dependent hysteresis nonlinearities," *Physica B: Condensed Matter*, vol. 407, no. 9, pp. 1365-1367, 2012.