# Tracking Control of a Piezoelectric Actuator Using a Bounded-Input Adaptive Backstepping Scheme and Sliding-Mode Observer

Juan Antonio ESCARENO, Didace HABINEZA
and Micky RAKOTONDRABE*

*Abstract*— The present paper deals with the motion control of a piezoelectric cantilevered actuator for different frequencies regimes. The motion behavior of the piezoelectric cantilever is affected by two main parasitic effects, the hysteresis and creep effects, which are considered as a generalized disturbance. Additionally, cantilevers's position is the only available measured state. The latter operational scenario reveals that the trajectory-tracking or regulation problem is a non-obvious task. Prior to the controller design, we have focused on the estimation of the velocity using a sliding-mode observer (SMO) in order to avoid the numerical derivative. On the other hand, the control design takes into account the limited response of the control input (saturation) and it is obtained using Backstepping framework assuring asymptotic stability of desired origin. Numerical simulations are first presented to verify the efficiency of the observer and controller algorithm. This shows satisfactory performance to fulfilling the motion control objective, regulation and tracking. Then experimental results were performed and validate the predicted performances.

## I. INTRODUCTION

The significant advance of microrobotics in recent years, from technological and theoretical point of view, has increased the application range in many fields. In the biomedical domain, the manipulation of biological agents (red blood cells or DNA) relies on piezoelectric-based microgrippers. In the field of micro-assembly, a key application is the development of MEMS (Micro Electro-Mechanical Systems) which themselves impact other areas as automotive, aeronautics and aerospace.

The attractive properties of piezoelectric actuators, with cantilevered structure in this case (called piezocantilever), which are the high resolution, the significant stiffness, and the large bandwidth exhibit great potential for applications like the manipulation of micro-sized objects. However, these active materials are constrained by adverse effects which are the hysteresis and the creep. Furthermore, it is important to underline that control systems, in practice, feature an amplitude-limited response. This aspect must be considered in the control design for stability and performance purposes. Hence, it is necessary to develop appropriate control designs that encompasses disturbance rejection and bounded control inputs.

Controlling systems featuring hysteresis have been addressed using feedforward and feedback control strategies, or a combination of both. The former relies on different models

The authors are with the FEMTO-ST Institute, UMR CNRS - UFC / ENSMM / UTBM, Automatic Control and Micro-Mechatronic Deparment, 24, rue Alain Savary, Besancon, France
*corresponding author: mrakoton@femto-st.fr

to compensate for the adverse effects of the hysteresis, while the latter use robust/adaptive techniques that disregard the knowledge of the hysteresis model.

In feedforward control of piezoelectric actuators, several approaches are available to model and then to compensate for the hysteresis: the Preisach [1] [2] [3] and the Prandtl-Ishlinskii approaches [4][5][6]. In both, a complex hysteresis is modeled by the sum of many basic hysteresis (hysterons). Both approaches can be very accurate subject to the use of a high number of elementary hysteresis, which represents a computational burden implementation. Alternatively, the Bouc-Wen model of hysteresis, initially proposed by Bouc [7] and further modified by Wen [8], has an interesting simplicity and is able to represent a large class of hysteresis. Although the low cost and the high packageability (no sensors required) of the used feedforward control approaches, their main limitation is the lack of robustness face to model uncertainties and to external disturbances.

Concerning feedback control, several approaches have been used. They include linear robust control techniques based on $H_\infty$ [9] [10], intervals based robust cotrol techniques [11], sliding mode control (SMC) and adaptive schemes [12][13][14][15] and state-feedback techniques [16][17]. Since the only measurement available in piezoelectric cantilevered actuator is the displacement (deflection of the cantilever), state-feedback control techniques required the use of observers. For this aim, [16] proposed a Kalman filtering computed to estimate non-measured states as well as for noise reduction in the measurement. In [18] an inverse-dynamics-based unknown input observer (UIO) was proposed to estimate the state vector as well as the force, considered as the external unknown input (disturbance). While the first observer was limited to linear model and then did not account for the hysteresis and creep of the actuator, the second observer was typified by a complex implementation. Furthermore, an essential point that is not considered by the above works is the bounded input control of the actuator. Indeed, if the voltage amplitude is too high, there is a risk of depolarization of the material leading to a loss of the piezoelectric properties. Consequently, overvoltages due to an excessive sollicitation on the piezoelectric actuator to effectuate rapid responses (large bandwidth) or large courses may destroy the latter.

The actual paper focuses on the motion control problem of a piezoelectric cantilever actuator (piezocantilever) operating under different references trajectories. The herein designed controller takes into account the fact the system features

a bounded-disturbed input. Also, given the fact that the only measured state is the piezocantilever's displacement, a sliding mode observer is employed to estimate the state vector and then used in the feedback control to ensure certain given performances for the closed-loop actuator. In the literature, several nonlinear tools have been introduced for analyzing and controlling linear and nonlinear systems with bounded inputs [23][24][25][26]. The control strategy presented in [26] has inspired us to propose a control strategy based on the backstepping technique because this considers not only bounded feedbacks but also bounded virtual controllers. More precisely, the developed controller considers the boundedness of the control inputs and it results in a simple and explicit expressions which are suitable for implementation. The proposed observer/control system is first verified by simulation which validates its efficiency. Then, an application to a home-made piezoelectric actuator with cantilevered structure was carried out and the experimental results validated the proposed approach.

The remainder of the paper is as follows. The description and model of the piezocantilever is presented in section-II. In section-III, the disturbance observer based on the UIO Technique as well as the control strategy to solve the tracking problem of the piezocantilever motion are detailed. Simulation results are presented in section-IV and experimental tests in section-V. Finally, the conclusions and perspectives are given in section-VI.

## II. MODEL OF THE PIEZOCANTILEVER

Without loss of generality relative to the modeling and control of piezoelectric actuators, the experimental test we will do in this paper and presented in section-V is based on a piezocantilever. Piezocantilevers are largely used in the development of microgripper and micromanipulators in microassembly, micromanipulation and microrobotics applications [19]. A piezocantilever, pictured in Fig.1-a, is a cantilever composed of two or several layers where at least one layer is based on piezoelectric material (piezolayer). The non-piezoelectric layers, called passive layer, is often a metal (copper, chrome...). When a voltage $u$ is applied to the piezolayer, it contracts or expands. Due to the interface constrain between the different layers and the difference on their contraction/elongation, the whole cantilever performs a bending $\delta$ (Fig.1-b). In the figure, $F$ is an external force applied to the piezocantilever if this latter is in interaction with the environment (objects...).

The dynamic and nonlinear behavior of a piezocantilever can be approximated by a second-order system, written as [20]:

$$a\ddot{\delta} + b\dot{\delta} + \delta = d_p u - h + c \qquad (1)$$

where $h$ and $c$ denotes the hysteresis and creep contributions respectively, while $a$, $b$ and $d_p$ are positive coefficients obtained from an identification process.

In the model (1), the hysteresis contribution $h(t)$ has been modelled with the quadrilateral technique [20], the
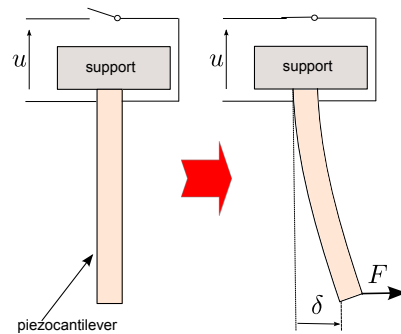


Fig. 1.   (a) and (b): principle of a piezocantilevered actuator.

Bouc-Wen equation [21], the Prandtl-Ishlinskii [22] or other techniques. This model (1) follows the Hammerstein structure which consists in cascading a linear dynamics with a nonlinear static part. Such structure is widely used to the modeling of hysteresis in piezoelectric systems. The creep contribution $c(t)$ has been considered as virtual external disturbance in [20], modelled with a low bandwidth linear dynamics in [22], and not accounted in many works.

The approach used in this paper consists in taking the hysteresis and creep as a single time-varying disturbance $\theta(t) = -h(t) + c(t)$. Furthermore, considering that the input voltage $u(t)$ should be limited in order to avoid the depolarization of the piezoelectric material, let us introduce the saturation operator for the control input of the actuator:

$$sat(u(t)) = \begin{cases} +U; & for \quad u(t) > +U \\ u(t); & for \quad -U \le u(t) \le +U \\ -U; & for \quad u(t) < -U \end{cases} \qquad (2)$$

where $U > 0$ is the saturation value.
Therefore, the model (1) becomes:

$$a\ddot{\delta} + b\dot{\delta} + \delta = d_p sat(u(t)) + \theta \qquad (3)$$

In general, there is no prior information about the disturbance dynamics, hence it is logical to suppose first that

$$\dot{\theta} \approx 0 \qquad (4)$$

(4) implies that the disturbance is a slow time-varying with respect to estimation provided by the adaptive control law. Although, we will illustrate by simulation that the adaptive estimation is able to track some fast time-varying disturbances.

## III. CONTROL STRATEGY

The problem addressed in this paper consists in fulfilling the tracking control objective of the piezoelectric actuator (for different trajectories and at different operational motion rates) with only the position as available measurement and with a saturated control input affected by an unknown-bounded disturbance (hysteresis and creep). However, to implement an efficient tracking controller, it is required to have a twice-differentiable reference vector $x^d = (\delta^d, \dot{\delta}^d)$, a full knowledge of the state vector $x = (\delta, \dot{\delta})^T$ and, if

it is possible, the knowledge of the disturbance term. For these aims, we first study an observer based on the sliding mode. This permits to estimate the states without having numerical differentiation. Then, a backstepping control scheme is proposed. This ensures a robustness of the closed-loop although the uncertainties on the disturbance term (related to the hysteresis and creep nonlinearities) and although the saturated input control.

*A. Sliding-Mode Observer*

The modified super-twisting observer scheme is used to reconstruct the state vector [27]. Such observer has several appealing features: (i) the observer can be designed separately from the controller, (ii) it is insensitive to unknown inputs, and (iii) it has a finite-time to converge to the state-vector.

The form of the observer is given by

$$\begin{aligned}
\dot{\hat{x}}_1 &= \hat{x}_2 + z_1 \\
\dot{\hat{x}}_2 &= f(t, x_1, \hat{x}_2, u) + z_2
\end{aligned} \tag{5}$$

where

$$\begin{aligned}
z_1 &= \beta |x_1 - \hat{x}_1|^{\frac{1}{2}} \mathrm{sign}(x_1 - \hat{x}_1) \\
z_2 &= \alpha \, \mathrm{sign}(x_1 - \hat{x}_1)
\end{aligned} \tag{6}$$

for our case $x_1 = \delta$, $x_2 = \dot{\delta}$ and

$$f(t, x_1, \hat{x}_2, u) = \tfrac{1}{a}(-b\hat{x}_2 - x_1 + d_p u(t)) \tag{7}$$

$\alpha > 0$ and $\beta > 0$ are the observer gains.

*B. Bounded-Input Robust Backsteppig*

The control objective is to design an amplitude-limited control to solve the tracking problem of the piezoelectric actuator, as well as to provide the corresponding stability analysis. To this end, the Backstepping technique provides an appropriate framework. Furthermore, notice that the aforementioned dynamic model (3) features a cascade form (lower-triangular) suitable to apply the Backstepping technique. Remind the model by assuming that the voltage $u(t)$ is within the limitation:

$$a\ddot{\delta} + b\dot{\delta} + \delta = d_p u(t) + \theta \tag{8}$$

Since we are concerned on solving the trajectory tracking problem, the latter differential model is rewritten into state-space model about the error variable $\xi$:

$$\begin{aligned}
\dot{\xi}_1 &= \xi_2 \\
\dot{\xi}_2 &= \tfrac{1}{a}\left[ d_p u(t) + \theta - b\dot{\delta} - \delta \right] - \ddot{\delta}^d
\end{aligned} \tag{9}$$

where $\xi_1 = \delta - \delta_d$ and $\xi_2 = \dot{\delta} - \dot{\delta}_d$ are the position and velocity errors, respectively.

For the sake of ease of writing, let us use the following notation simplification:

$$\tanh(\chi) = \sigma(\chi) \tag{10}$$

Before proceeding with the control design, let us present the following useful properties

- **P1.** $(1 - \sigma^2(\chi)) \leq 1$
- **P2.** $\sigma(\chi) \leq \chi$

- **P3.** $\dot{\sigma}(\chi) = \left[ 1 - \sigma^2(\chi) \right] \dot{\chi}$

**Step 1:** Let us propose the Candidate Lyapunov Function (CLF) to deduce a control that stabilizes the first integrator subsystem (9a)

$$\mathcal{V}_1(\xi_1) = \frac{1}{2}\xi_1^2 \tag{11}$$

whose time-derivative is give by

$$\dot{\mathcal{V}}_1(\xi) = \xi_1 \xi_2 \tag{12}$$

which is rendered negative-definite ($\dot{\mathcal{V}}_1(\xi) < 0$) if we consider $\xi_2$ as virtual (bounded) controller. Thus, we take:

$$\xi_2 = -\lambda_1 \sigma(\xi_1) \tag{13}$$

where $\lambda_1$ is a positive scalar constant. Hence, we can conclude that $\xi_1$ is not only stable but also converges asymptotically to the origin.

**Step 2:** Let us propose a bounded error state variable, where the previous virtual controller is used as a reference in order to impose a constrained behavior to $\xi_2$, i.e. $\xi_2^d = -\lambda_1 \sigma(\xi_1)$. We have:

$$\sigma(z) = \xi_2 - \xi_2^d = \xi_2 + \lambda_1 \sigma(\xi_1) \tag{14}$$

which produces the bounded extended state

$$\xi_2 = \sigma(z) - \lambda_1 \sigma(\xi_1) \tag{15}$$

Differentiating (14) with respect to time and using (15) yield

$$(1 - \sigma^2(z))\dot{z} = \dot{\xi}_2 + \lambda_1 \left[ 1 - \sigma^2(\xi_1) \right] \left[ \sigma(z) - \lambda_1 \sigma(\xi_1) \right] \tag{16}$$

Let the final CLF be

$$\mathcal{V}_2(\xi_1, z) = \frac{1}{2}\xi_1^2 + \frac{1}{2}\sigma^2(z) + \frac{1}{2\gamma}\tilde{\theta}^2 \tag{17}$$

where $\tilde{\theta} = \theta - \hat{\theta}$ represents the estimation error. The corresponding time-derivative of $\mathcal{V}_2(\xi_1, z)$ is

$$\dot{\mathcal{V}}_2(\xi_1, z) = -\lambda_1 \xi_1 \sigma(\xi_1) + \xi_1 \sigma(z) + \sigma(z)(1 - \sigma^2(z))\dot{z} \tag{18}$$

Replacing (16) in (18), and considering **P1** yield:

$$\dot{\mathcal{V}}_2(\xi_1, z) \leq -\lambda_1 \xi_1 \sigma(\xi_1) + \xi_1 \sigma(z) + \sigma(z) \left[ \dot{\xi}_2 + \lambda_1 (\sigma(z) - \lambda_1 \sigma(\xi_1)) \right] \tag{19}$$

Remind that:

$$\dot{\xi}_2 = \frac{1}{a}(d_p u(t) + \theta - b\dot{\delta} - \delta) - \ddot{\delta}^d \tag{20}$$

Therefore, in order to render (19) into a negative-definite function, we introduce the following bounded controller in (20):

$$\begin{aligned}
u = \\
\frac{1}{d_p}\left[ \delta + b\dot{\delta} - \hat{\theta} + a\ddot{\delta}^d - a\lambda_1 \left( \sigma(z) + \lambda_1 \sigma(\xi_1) \right) - a\lambda_2 \sigma(z) \right]
\end{aligned} \tag{21}$$

and the adaptation law

$$\dot{\hat{\theta}} = \gamma \sigma(z) \tag{22}$$

This leads us to:

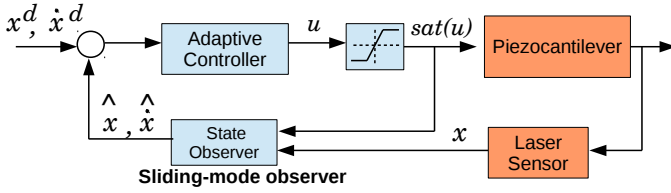$$\dot{\mathcal{V}}_2(\xi_1, z) = -\lambda \xi_1 \sigma(\xi_1) + \xi_1 \sigma(z) - \lambda_2 \sigma^2(z) \tag{23}$$

Fig. 2. Block diagram including the observer and the controller.

| Parameter | Value[units] |
|-----------|--------------|
| $d_p$ | 6.8 [$\mu$m/V] |
| $A_{bw}$ | 3.08[$\mu$m/V] |
| $B_{bw}$ | 0.2556[$V^{-1}$] |
| $C_{bw}$ | 0.0411[$V^{-1}$] |

TABLE I

BOUC-WEN MODEL PARAMETERS

which verifies

$$\dot{\mathcal{V}}_2(\xi_1, z) \leq -\lambda\sigma^2(\xi_1) + \xi_1\sigma(z) - \lambda_2\sigma^2(z) \qquad (24)$$

In order to draw conclusion about the stability, let us rewrite the previous equation into a vectorial form:

$$\dot{\mathcal{V}}_2(\nu) = -\sigma(\nu)^T A \sigma(\nu) \qquad (25)$$

with $\sigma(\nu) = (\sigma(\xi_1), \sigma(z))^T$. Hence:

$$\dot{\mathcal{V}}_2(\nu) = -\begin{pmatrix} \sigma(\xi_1) \\ \sigma(z) \end{pmatrix}^T \begin{pmatrix} \lambda_1 & -\frac{1}{2} \\ -\frac{1}{2} & \lambda_2 \end{pmatrix} \begin{pmatrix} \sigma(\xi_1) \\ \sigma(z) \end{pmatrix} \qquad (26)$$

is negative-definite if $A > 0$, which holds for $\lambda_1\lambda_2 > \frac{1}{4}$. Thus, $\nu$ converges asymptotically to the origin. This implies that $\mathcal{V}_2 \to 0$ as $t \to \infty$. Hence, it follows that $\xi_1 \to 0$ and $\sigma(z) \to 0$, which implies that $\xi_2 \to \xi_2^d$ according to (14) fulfilling the tracking objective.

## IV. NUMERICAL SIMULATIONS

The aim of this section is to evaluate the performances of the proposed observer and control approach for the disturbed system in equation (8) at simulation level. Notice that the simulation permits to validate both the observer and the control in details while the experimental tests of the next section only permits to analyze the results of the control. Indeed, there is no sensor to measure the velocity and then to discuss more in details on the efficiency of the observer. For the simulation, a hysteresis model that tracks $h(t)$ of equation (1) is required. For that, we proposes the Bouc-Wen model for which the hysteresis $h(t)$ is given by:

$$\dot{h} = d_p A_{bw}\dot{u} - B_{bw}|\dot{u}|h - C_{bw}\dot{u}|h| \qquad (27)$$

where $A_{bw}$, $B_{bw}$ and $C_{bw}$ are parameters that control the shapes of the hysteresis. These parameters have been identified using the least-square algorithm by using experimental hysteresis data from the piezocantilever that will be used for the experiments in the next section. The identified parameters are illustrated in table-I. On the other hand, an ARMAX approach (Autoregressive-moving-average model with exogenous inputs) has been employed to identify the parameters of the dynamics part of (8) with an experimental step response of the piezocantilever. We obtain: $a = 4.4209 \times 10^{-9}$ and $b = 3.7378 \times^{-6}$.

Fig.2 depicts the block diagram of the controller and of the observer.

### A. Observer performances

In this case, the state vector is partially known: position is a measured state and velocity is reconstructed. The performances, in open-loop, of the sliding-mode observer are depicted on Fig.3 where it can be observed that estimated position and velocity satisfactorily fulfill the desired performances objective. In particular, Fig.3-c and d show the quick convergence of the estimate position and of the estimate velocity to the actual position and actual velocity respectively. Indeed, the convergence times for both observation are less than $6ms$.

### B. Control performances

The performances of the controlled piezoelectric cantilever (with observer and controller) while tracking three sine trajectories (amplitude: $50\mu m$ and frequency: $1Hz$, $10Hz$ and $30Hz$) are depicted in figures Fig.4 (for $f = 1Hz$), Fig.5 (for $f = 10Hz$) and Fig.6 (for $f = 30Hz$). From Fig.4-a and c, Fig.5-a and c and Fig.6-a and c, it is seen that that the error for the position tracking is less than $1\%$ at stationary regime. Concerning the velocity tracking, we see from Fig.4-b and d, Fig.5-b and d and Fig.6-b and d that the error is negligible. To summarize, for low or high frequencies of the reference, the control strategy ensures a satisfactorily behavior although we are using a reconstructed velocity
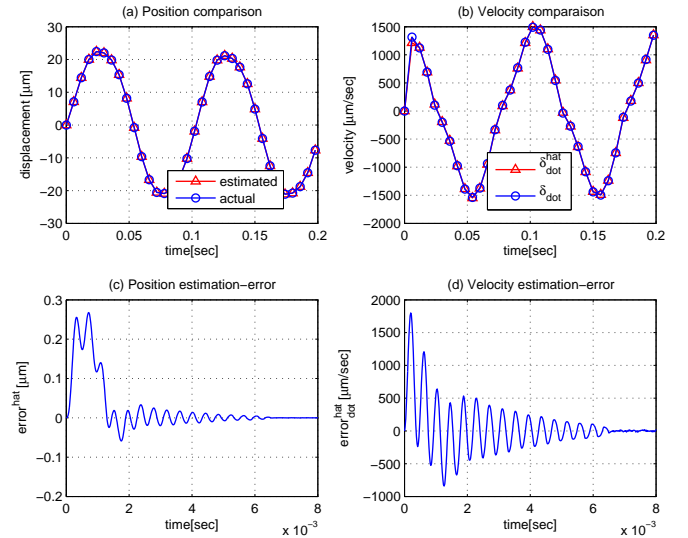


Fig. 3. Observer performances in open loop. (a): actual position and estimate position. (b): actual velocity and estimate velocity. (c): error on position. (d): error on velocity.

state and that the disturbance was assumed with a slow-time varying dynamics in the synthesis. The observer gains and control gains used for the numerical simulations are depicted in table-II. These values were calculated manually in order to satisfy the conditions given in section-III. Recall that the controller gains are multiplied by the dynamic parameter $a = 4.4209 \times 10^{-9}$ as depicted in equation (21).

| Observer gains | f=1Hz | f=10Hz | f=30Hz |
|---|---|---|---|
| $\beta$ | $1.2 \times 10^2$ | $20 \times 10^2$ | $60 \times 10^2$ |
| $\alpha$ | $2 \times 10^3$ | $90 \times 10^4$ | $580 \times 10^4$ |
| **Control gains** | **f=1Hz** | **f=10Hz** | **f=30Hz** |
| $\lambda_1 = \lambda_2$ | $15 \times 10^2$ | $30 \times 10^2$ | $65 \times 10^2$ |
| $\gamma$ | 0.3 | 0.3 | 0.4 |

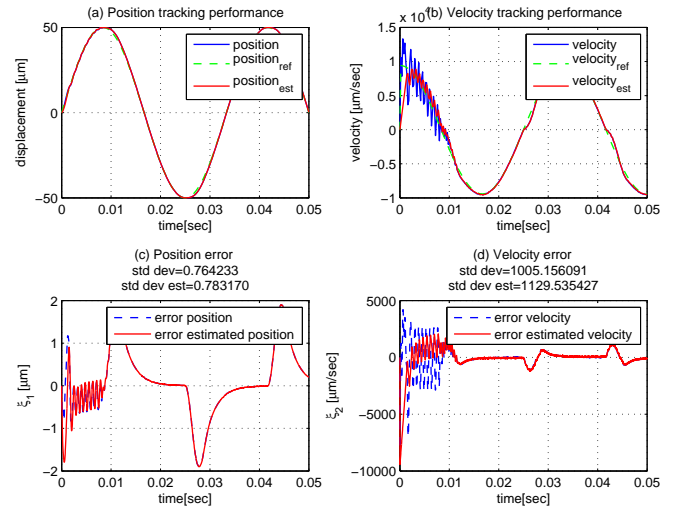TABLE II

OBSERVER AND CONTROL GAINS
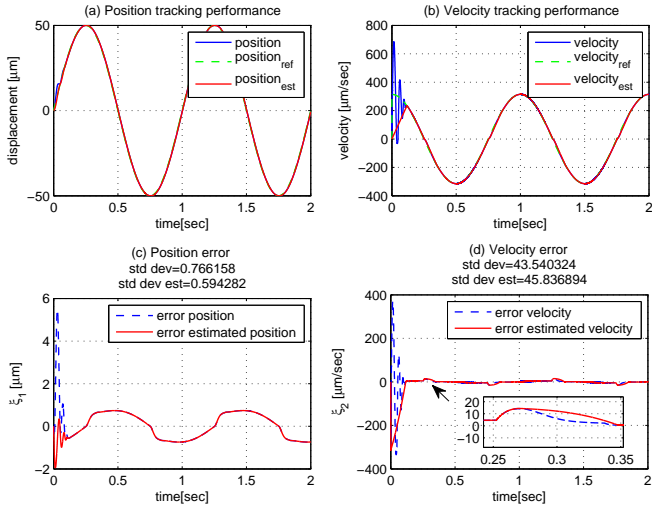


Fig. 6. Control Input performance.



Fig. 4. Trajectory tracking performance with hysteresis compensation.
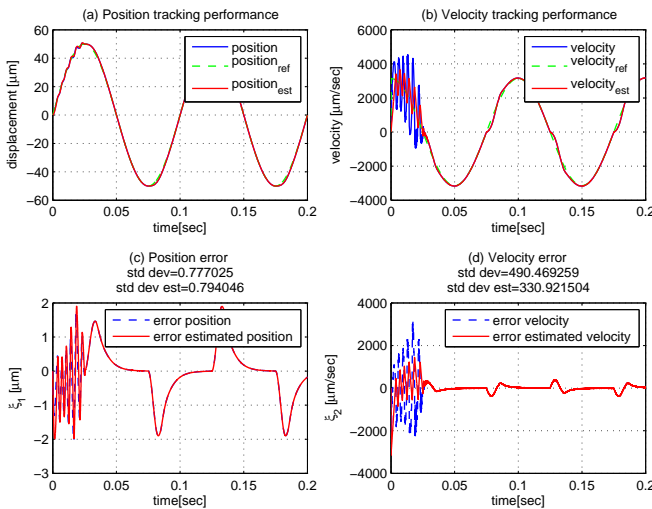


Fig. 5. Observer performance.

## V. EXPERIMENTAL TESTS

In this section, the previous observer and control strategy are implemented and applied to a piezocantilever as experimental tests. The setup, presented in Fig.7. This is composed of:

- the piezocantilever has 36 layers and have active dimensions (active length $\times$ width $\times$ total thickness) of: $15mm \times 2mm \times 2mm$. All layers are based on the lead zirconate titanate (PZT) material. This piezocantilever can bends along two directions ($y$ and $z$) but we experiment our observer and control strategy in this paper with the $y$ axis. This bending along $y$ is called $\delta$.
- an optical sensor LC2420 from *Keyence* Company that is used to measure the bending (position, or displacement) $\delta$ of the tip of the piezocantilever. The sensor is set to have a resolution of $50nm$, an accuracy of $200nm$ and a bandwidth in excess of $1500kHz$.
- and a computer and a dSPACE board for the implementation of the observer and the controller and for the management of the different signals. The freshing time is set equal to $0.2ms$ which largely considers the dynamics of the actuator.

The fact that the piezocantilever has several layers permits to work with low voltage (between $-10V$ and $+10V$) to obtain sufficient displacement (up to $50\mu m$). This limitation of the control voltage should be respected, otherwise the electrical field $\frac{u}{e_{pzt}}$ (with $e_{pzt}$ being the thickness of each layer) may be too excessive and results in the depolarization of the layers. The saturation operation considered by the control strategy developed in section-III is therefore essential for this application. The observer gains and control gains used for the implementation are that of $f = 30Hz$ in table-II.

The first experimental test consists in applying a step input reference $\delta^d = 50\mu m$ to the closed-loop. The step response is pictured in Fig.8. The result indicates a negligible tracking
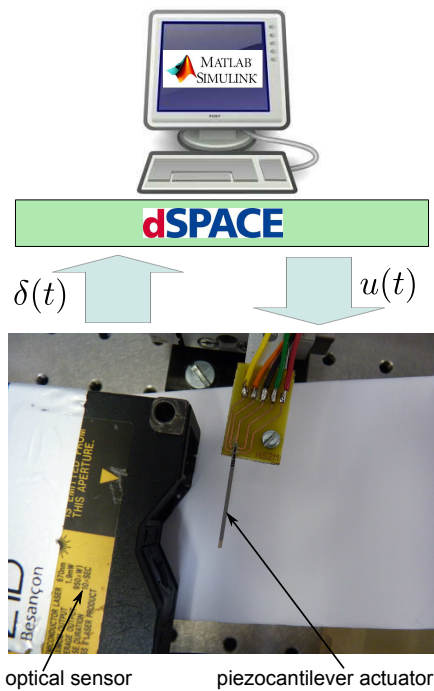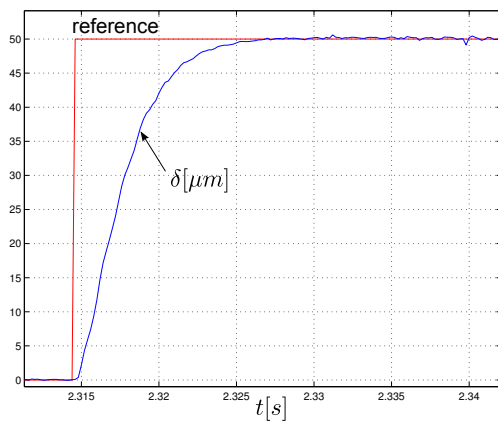
Fig. 7. Presentation of the setup.
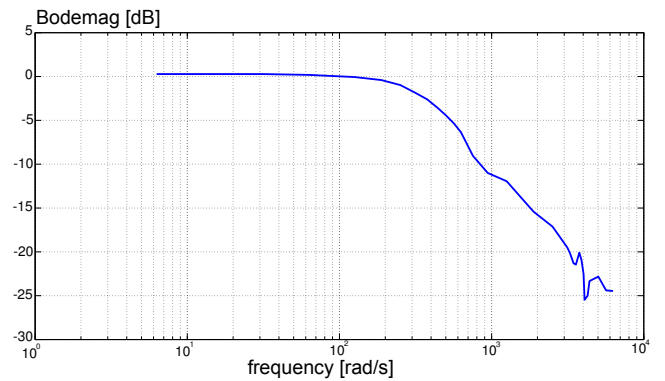


Fig. 8. Experimental step response.



Fig. 9. Experimental harmonic response.

state vector of the piezocantilever while having as only measurement the position. The estimated state vector, composed of the position and the velocity, is used in an adaptive control law to counteract the adverse effects provoked by the hysteresis and the creep which were considered as disturbance. Since real-world systems feature a finite response (saturation), we have designed a saturation-based controller via the Backstepping technique considering into account bounded states and bounded virtual controllers throughout the control design. The asympotic stability of the controller was proven and the convergence of the states to the origin was guaranteed with the proposed control strategy. Simulation analysis was first carried out with the observer and control strategy in order to validate the efficiency of the proposed approach. Then experimental tests with a piezocantilever actuator having 36 layers were performed in order to confirm the theory and the simulation results. Both the simulation and the experimental results demonstrated the efficiency and the performances proposed by the observer and the control strategy.

error of the controlled system and a step response about $7.5ms$.

In the second test, a harmonic analysis is carried out. It consists in applying a sine reference input to the closed-loop at different frequencies ranging between $1Hz$ ($6.283rad/s$) and $1kHz$ ($6283rad/s$). The resulting magnitude is afterwards plotted (see Fig.9). As demonstrated by the result, the bandwidth of the closed-loop is in excess of $72Hz$ ($450rad/s$) which is higher than that expected in simulation ($30Hz$).

## VI. CONCLUDING REMARKS

In this paper the tracking control problem was addressed for a piezocantilever actuator. To deal with such problem, we have used a sliding-mode observer to estimate the complete

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